Sinusoidal functions (sec. 7.5)

Thursday, November 19, 2020 3:00 PM

Overview of transformations:

$|B| > 1$ is a horizontal compression

$|B| < 1$ is a horizontal stretch

horizontal shift

vertical shift

$$y = Af(B(t-h)) + k$$

- Stretch by a factor of $|A| > 1$
- compression by a factor of A ie $|A| < 1$.

$$y = A\sin(B(t-h)) + k$$

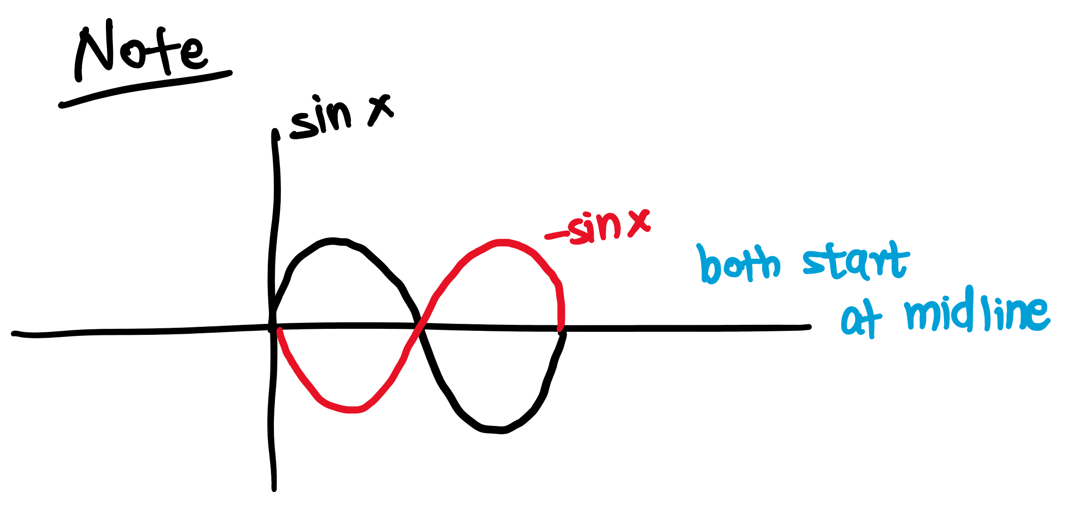
$$y = A\cos(B(t-h)) + k$$

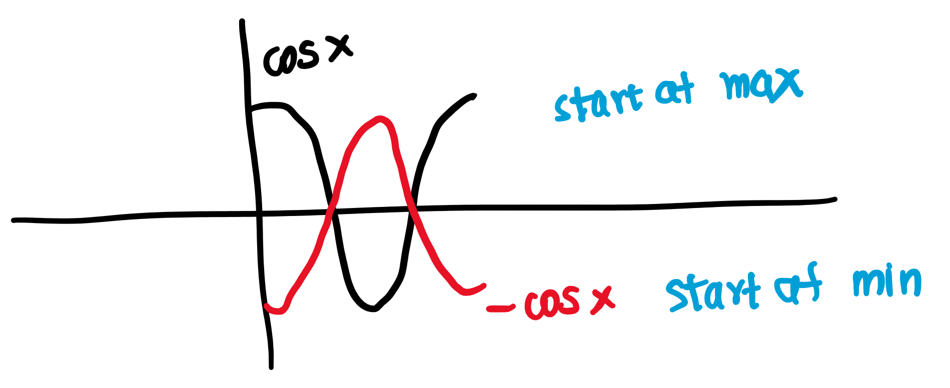
amplitude $= \frac{\max - \min}{2}$

$\frac{2\pi}{\text{new period}}$

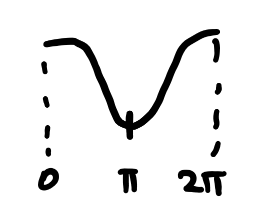
midline $= \frac{\max + \min}{2}$

Note

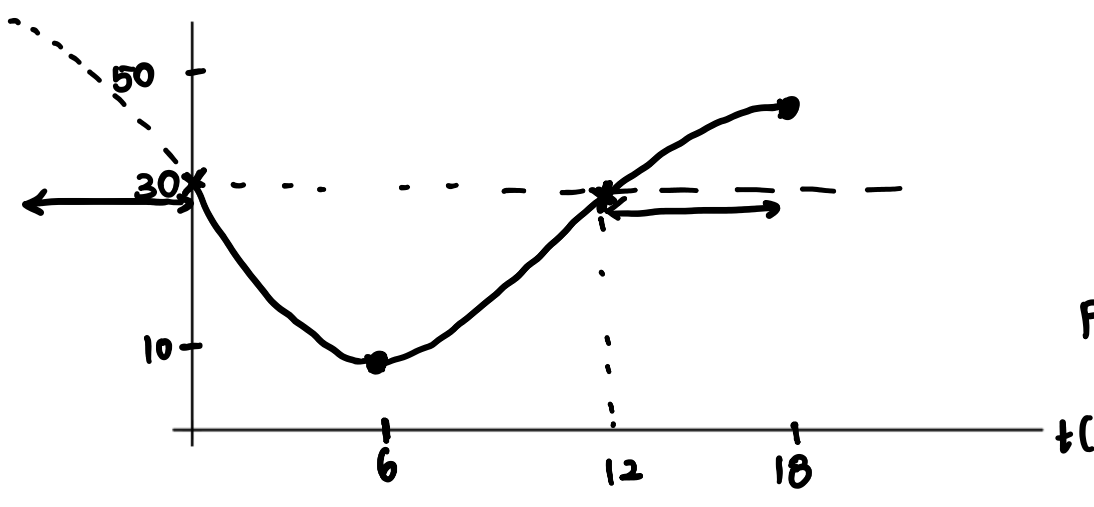




5. [13 points] Jordan owns a 24-hour coffee shop. The coffee brewing rate (or CBR) at Jordan's coffee shop varies throughout the day. The CBR is highest at 6 AM, when coffee is brewed at a rate of 50 pounds of coffee per hour. It is lowest at 6 PM, when coffee is brewed at a rate of only 10 pounds of coffee per hour. Suppose that t hours after noon, the CBR, in pounds of coffee per hour, of Jordan's coffee shop can be modeled by a sinusoidal function $C(t)$ with period 24 hours.



a. [4 points] On the axes provided below, sketch a well-labeled graph of $C(t)$ for $0 \le t \le 24$.



midline $= \frac{\max + \min}{2} = \frac{50+10}{2} = 30$

period $= 2 \times (18-6) = 2(12) = 24$

b. [4 points] Find a formula for $C(t)$.

$$y = -20\sin\left(\frac{\pi}{12}t\right) + 30$$

amplitude $= \frac{\max - \min}{2} = \frac{50-10}{2} = 20$

Check: $t = 6$

$y = -20\sin\left(\frac{\pi}{2}\right) + 30 = -20 + 30 = 10$ ✓

$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{24} = \frac{\pi}{12}$

Answer: $C(t) =$ ______

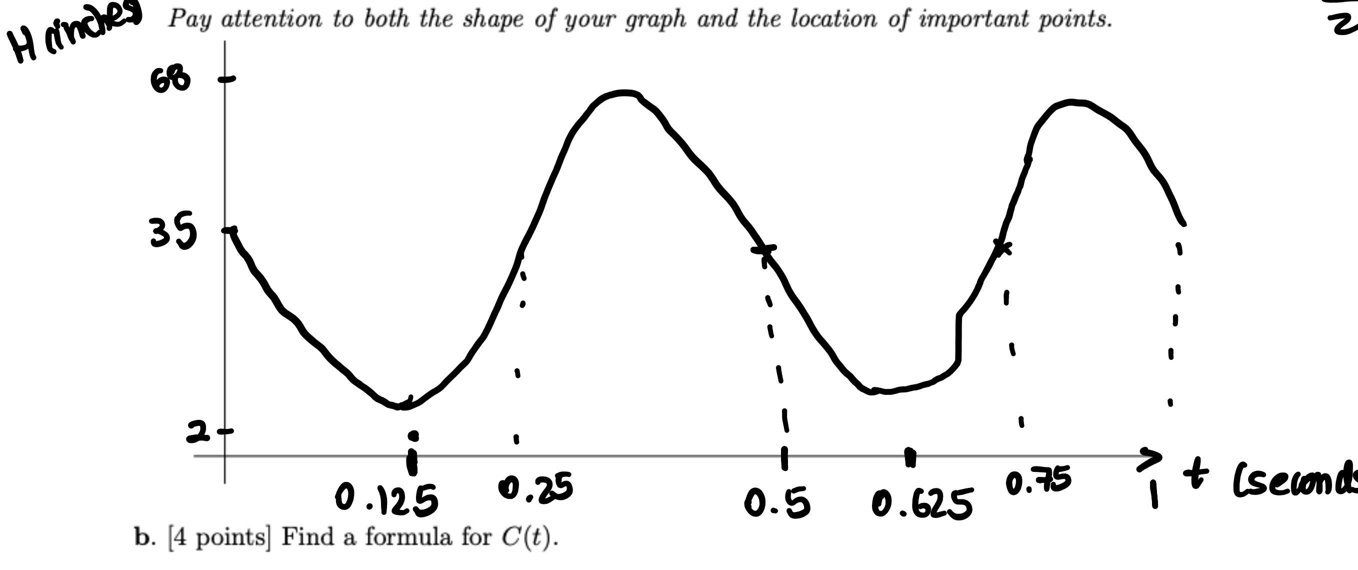
c. [5 points] For how many hours each day is the CBR of Jordan's shop at least 40 pounds of coffee per hour? *Remember to show your work.*

Answer: ______

9. [13 points] Cal is jumping a rope being swung by Gen and Algie while Maddy runs a stopwatch. There is a piece of tape around the middle of the rope. When the rope is at its lowest, the piece of tape is 2 inches above the ground, and when the rope is at its highest, the piece of tape is 68 inches above the ground. The rope makes two complete revolutions every second. When Maddy starts her stopwatch, the piece of tape is halfway between its highest and lowest points and moving downward. The height H (in inches above the ground) of the piece of tape can be modeled by a sinusoidal function $C(t)$, where t is the number of seconds displayed on Maddy's stopwatch.

midline $= \frac{68+2}{2} = \frac{70}{2} = 35$

a. [4 points] On the axes provided below, sketch a well-labeled graph of two periods of $C(t)$ beginning at $t = 0$.
Pay attention to both the shape of your graph and the location of important points.



b. [4 points] Find a formula for $C(t)$.

ampl. $= 68 - 35 = 33$

$\frac{2\pi}{0.5} = 4\pi$

Answer: $C(t) = -33\sin(4\pi t) + 35$

c. [5 points] Now Gen takes a turn at jumping while Cal and Algie swing the rope. Maddy resets the stopwatch and starts it over again. Let $G(w)$ be the height (in inches above the ground) of the piece of tape when Maddy's stopwatch says w seconds. A formula for $G(w)$ is

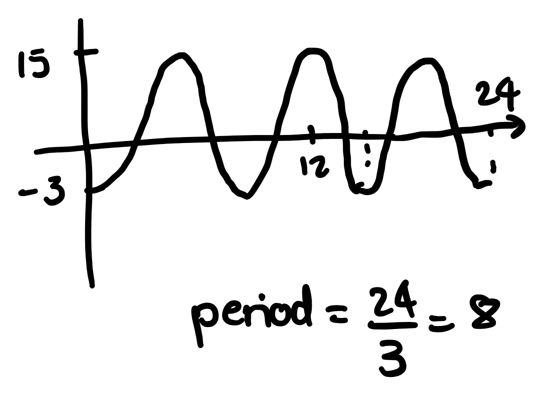
$$G(w) = 41 + 38\cos(2\pi w).$$

Maddy is 60 inches tall. For how long (in seconds) during each revolution of the rope is the piece of tape higher than the top of Maddy's head? (Assume Maddy is standing straight while watching the stopwatch.) *Remember to show your work.*

Answer: ______

6. [11 points] A company designs chambers whose interior temperature can be controlled. Their chambers come in two models: Model A and Model B.

a. [5 points] The temperature in Model A goes from its minimum temperature of −3°C to its maximum temperature of 15°C and returning to its minimum temperature three times each day. The temperature of this chamber at 10 am is 15°C. Let $A(t)$ be the temperature (in °C) inside this chamber t hours after midnight. Find a formula for $A(t)$ assuming it is a sinusoidal function.



period $= \frac{24}{3} = 8$

amplitude $= \frac{\max - \min}{2} = \frac{15-(-3)}{2} = \frac{18}{2} = 9$

period =

midline $= y = \frac{\max + \min}{2} = \frac{15-3}{2} = \frac{12}{2} = 6$

Answer: $A(t) = -9\cos\left(\frac{2\pi}{8}(t+2)\right) + 6$

b. [6 points] Let $B(t)$ be the temperature (in °C) inside Model B t hours after midnight, where

$$B(t) = 5 - 3\cos\left(\frac{3}{7}t + 1\right).$$

Find the two smallest positive values of t at which the temperature in the chamber is 6°C. Your answer must be found algebraically. *Show all your work and give your answers in exact form.*

Answer: $t =$ ______ and ______