Shifts

Thursday, October 22, 2020 7:40 PM

Shifts

Vertical shifts: If $f(x)$ is the original function then

$y = f(x)+k$ is the graph of $f(x)$ shifted up by k units if $k>0$.

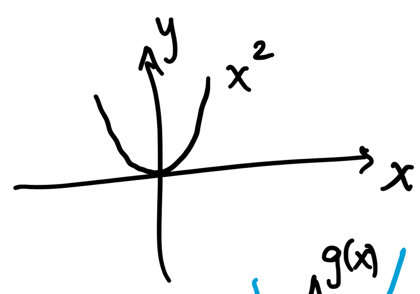
$y = f(x)-k$ is the graph of $f(x)$ shifted down by k units if $k>0$.

Horizontal shifts

$y = f(x+h)$ is the graph of $f(x)$ shifted to the left by h units if $h>0$

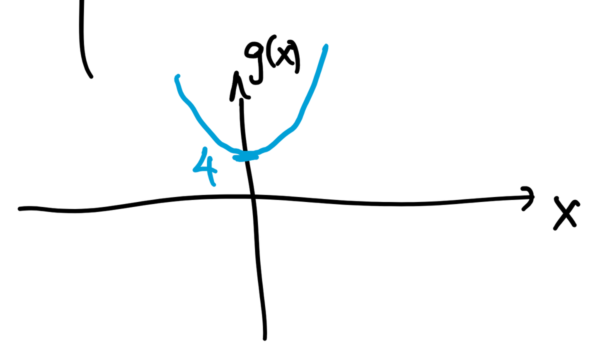
$y = f(x-h)$ is the graph of $f(x)$ shifted to the right by h units if $h>0$

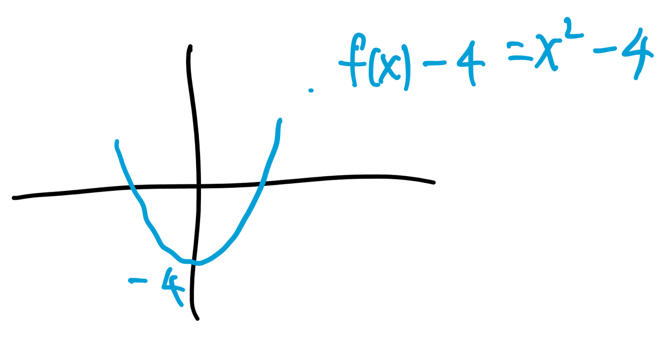
Example $y = x^2 = f(x)$



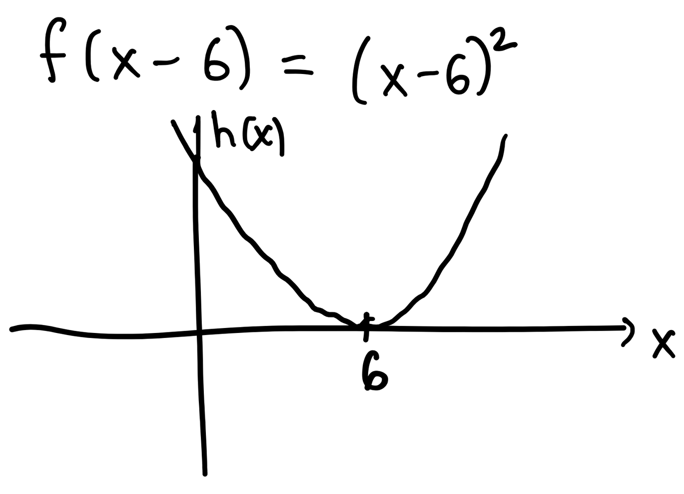
x-intercept: $x=0$
y-intercept: $y=0$

$g(x) = f(x)+4 = x^2+4$

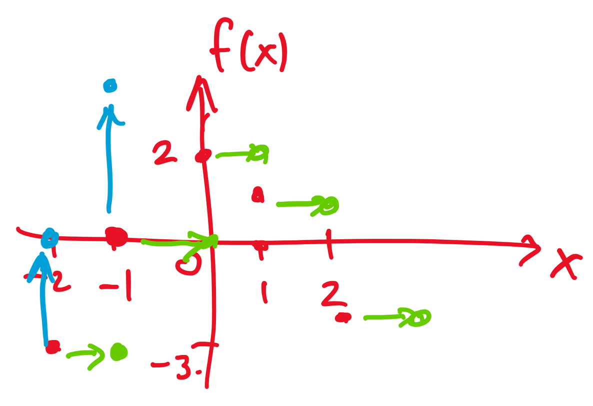




$h(x) = f(x-6) = (x-6)^2$



x-intercept: $(x-6)^2=0 \Rightarrow x=6$
y-intercept: $y=0$



Example (WebWork)

x	-2	-1	0	1	2
f(x)	-3	0	2	1	-1

x	-1	0	1	2	3
g(x)	-3	0	2	1	-1

x	-3	-2	-1	0	1
h(x)		0	3	5	4

x	-1	0	1	2	3
m(x)	0	3	5	4	2

Complete the following:

a) $g(x) = f(x-1)$ ← horizontal shift to the right by 1

b) $h(x) = f(x)+3$ vertical shift up by 3

c) $m(x) = f(x-1)+3$

$= g(x)+3$

Example You have that $f(x)$ contains the point $\boxed{(7,10)}$.

What is a point on the following transformed graphs.

(a) $f(x-5)$ $(12,10)$ only $x=7$ is affected since the shift is horizontal.

(b) $f(x)-7$ $(7,3)$ only $y=10$ is affected since the shift is vertical.

(c) $f(x+2)+7$ $(5,17)$ both a horizontal & a vertical shift

Example $f(x)$ is the original function w/ domain $-6<x<6$, range $-3<y<5$

What is the domain of $f(x-4)$: $-2<x<10$ (shift to the right by 4)

and the range : $-3<y<5$

Example Let $g(x)$ contains $(-7,8)$

Write a formula for the transformed function of g whose graph contains

(a) $(-2,8)$ $g(x-5)$

(b) $(-7,12)$ $g(x)+4$

Compositions and inverses

Let $f(x)$ and $g(x)$ be two functions, then $f(g(x))$ would be a composition of f with g. The function $f(g(x))$ is defined by using the output of g as the input of f.

eg. $f(x) = x^3+4x+5$ $\quad g(f(x)) = \sqrt{x^3+4x+5}$

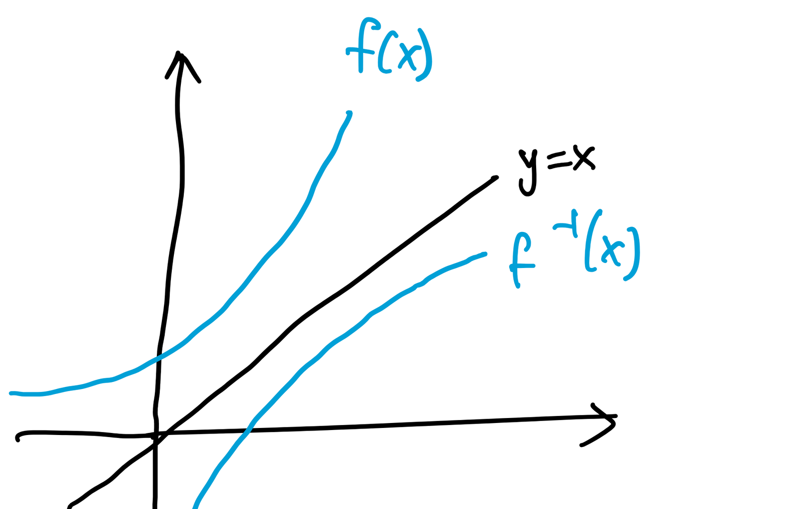
$g(x) = \sqrt{x}$ $\quad f(g(x)) = f(\sqrt{x})$

$= (\sqrt{x})^3 + 4\sqrt{x}+5$

$g(f(x)) \neq f(g(x))$

$f(x) = x^2$, $f^{-1}(x) = \sqrt{x}$, $f(f^{-1}(x)) = x$

$\rightarrow f^{-1}(f(x)) = x$



$f^{-1}(x)$ is the reflection of $f(x)$ along $y=x$.

$y = f(x) \Leftrightarrow f^{-1}(y) = f^{-1}(f(x))$

$f^{-1}(y) = x$

domain of $f(x)$ is the range of $f^{-1}(x)$

range of $f(x)$ is the domain of $f^{-1}(x)$.

Note Find the formula for an inverse:

$f(x) = \frac{x^2+5}{3}$

Step 1: Replace $f(x)$ with y

$y = \frac{x^2+5}{3}$

Step 2: Rearrange to write x in terms of y.

$3y = x^2+5$

$3y-5 = x^2$

$x = \sqrt{3y-5}$

Step 3: Replace y with x and x with $f^{-1}(x)$

$f^{-1}(x) = \sqrt{3x-5}$