Separation of variables

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Consider the same example $\frac{dy}{dx} = -\frac{x}{y}$.

How do you obtain that $x^2+y^2=C$ is the solution?

The method of separation of variables works by putting all the x-values on one side of the equation and all the y-values on the other

METHOD

Step 1 Separate the x's with the y's.

$$y\,dy = -x\,dx$$

Step 2 Integrate each side separately

$$\int y\,dy = \int -x\,dx$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + C$$

$$y^2 = -x^2 + 2C \qquad \text{let } k = 2C.$$

$$x^2+y^2=k \qquad \leftarrow \text{circles.}$$

NB

If you are given an initial condition of the form $y(A)=B$ you can use it to find the constant of integration.

Note A differential equation is called separable, if it can be written in the form

$$\frac{dy}{dx} = f(x)g(y)$$

1. Determine which of the following differential equations are separable. Do not solve the equations. Y = yes, N = no

(a) $y' = y$ Y (b) $y' = x + y$ N
(c) $y' = xy$ Y (d) $y' = \sin(x+y)$ N
(e) $y' - xy = 0$ Y (f) $y' = y/x$ Y
(g) $y' = \ln(xy)$ N (h) $y' = (\sin x)(\cos y)$ Y
(i) $y' = (\sin x)(\cos xy)$ N (j) $y' = x/y$
(k) $y' = 2x$ (l) $y' = (x+y)/(x+2y)$

$$\frac{dy}{dx} = xy$$

$$\frac{dy}{dx} = x+y$$

$$\frac{dy}{dx} - y = x$$

$$dy - y\,dx = x\,dx$$

$$\frac{dy}{dx} = \ln(xy)$$

$$e^{dy/dx} = xy$$

Example

$$B^2 + 2B\frac{dB}{dt} = 2500, \qquad B(0)=0$$

$$2B\frac{dB}{dt} = 2500 - B^2$$

$$\frac{dB}{dt} = \frac{2500-B^2}{2B} \leftarrow \text{divide by } \left(\frac{2500-B^2}{2B}\right)$$

$$\frac{2B}{2500-B^2}\frac{dB}{dt} = 1$$

$$\int \frac{2B}{2500-B^2}\,dB = \int dt$$

partial fractions or u-subst