Rational functions (sec. 11.4)

Monday, December 7, 2020 6:26 PM

Mastery 8 — closing on Dec 11
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<u>Definition</u>: A rational function $r(x)$ is the ratio of two polynomial functions $p(x)$ and $q(x)$, i.e. $\boxed{r(x) = \frac{p(x)}{q(x)}}$, where $q(x) \neq 0$.

For x large enough (positive or negative) the graph behaves like a power function.

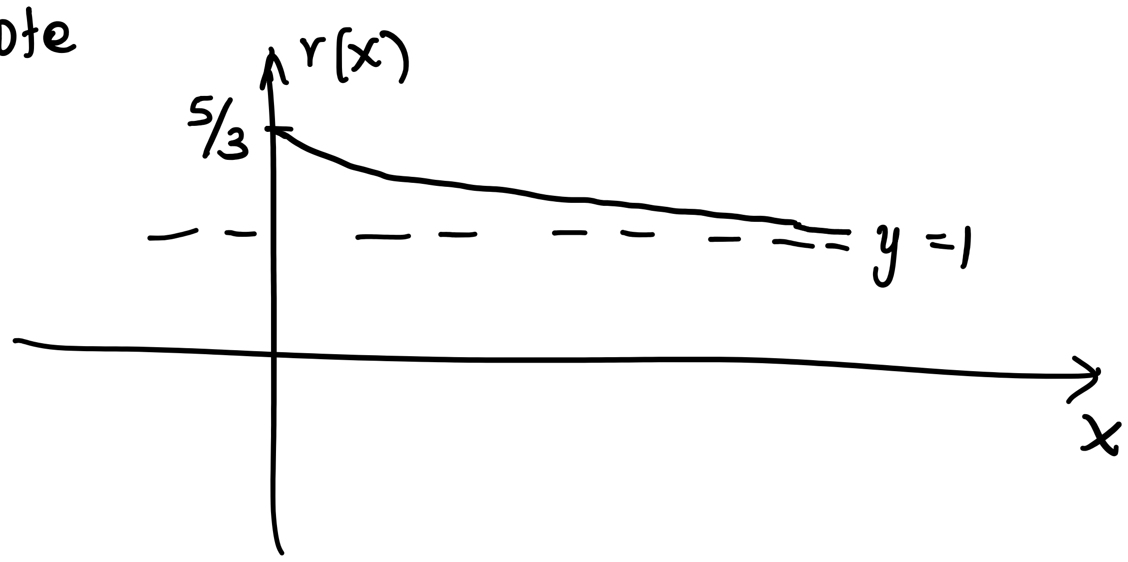
$$(\star) \quad \lim_{x \to \pm\infty} r(x) = \lim_{x \to \pm\infty} \frac{p(x)}{q(x)} = \lim_{x \to \pm\infty} \frac{\text{leading term of } p(x)}{\text{leading term of } q(x)}$$

Reminder, if $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, $a_n x^n$ would be the leading term.

If the limit $(\star)$ exists, it gives the horizontal asymptote of $r(x)$.

<u>e.g</u> Consider $r(x) = \frac{x+5}{x+3}$

$\lim_{x \to \infty} r(x) = \lim_{x \to \infty} \frac{x+5}{x+3} = \lim_{x \to \infty} \frac{x}{x} = 1$. Here $y=1$ is the horizontal asymptote



<u>Note</u> Polynomial graphs cannot level off to a horizontal line as the graphs of rational functions can.