Quadratic functions

Monday, October 26, 2020 5:24 PM

A parabola is the graph of a quadratic function defined by

$$\boxed{y = ax^2+bx+c}$$

where a, b, c are constants with $a \neq 0$.

You can also write it as $y = a(x-r)(x-s)$. ← factorized form / factored form

Here it's easy to find the zeros (find the x-values for which $y=0$)

$$0 = a(x-r)(x-s)$$

zeros: $x=r$ and $x=s$.

Quadratic formula: $y=ax^2+bx+c$

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

Example $y = x^2-x-6$

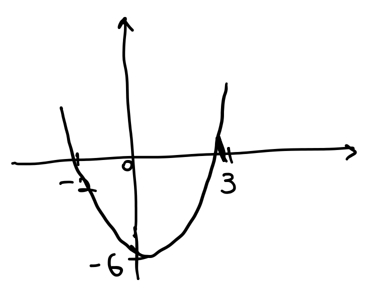
Factored form: $y = (x-3)(x+2)$

What are the zeros? $x=3$ and $x=-2$.

or use quadratic formula

$a=1$, $b=-1$, $c=-6$

$$x = \frac{1 \pm \sqrt{1-4(1)(-6)}}{2} = \frac{1 \pm \sqrt{25}}{2} = \frac{1 \pm 5}{2} \begin{cases} \frac{6}{2} = 3 \\ -\frac{4}{2} = -2 \end{cases}$$



Note

To determine the concavity of $y=ax^2+bx+c$ look at the sign of coefficient a:

- if $a>0$, the parabola is concave up
- if $a<0$, the parabola is concave down.

Example $y = -3x^2-3x+2$ concave down

$$y = -3(-2x+1)(x-5)$$
$$= -3(-2x^2+10x+x-5)$$
$$= 6x^2-33x+15 \quad \text{concave up}$$

8. [12 points] Top Norwegian skier, Wallace, is participating in the ski jumping world championship. During his practice jump, his height (in meters) above his landing spot on the ground t seconds after going airborne is given by $h = A(t) = -4.9t^2+15t+60$. standard.

Throughout this problem include **units**, and express your answers in **exact form** or round your answers to **three decimal places**. Be sure show all the steps needed to get your answers, and **circle** your final answer. Answers with no work shown will not receive credit.

a. [2 points] How high above his landing spot was Wallace when he first went airborne?

$t=0$ $h = A(0) = -4.9(0)^2+15(0)+60 = 60$ m

b. [4 points] At what time t does Wallace land on the ground?

$h=0 \Rightarrow 0 = -4.9t^2+15t+60$

quadratic formula:

$$t = \frac{-15 \pm \sqrt{15^2-4(-4.9)(60)}}{2(-4.9)}$$

$$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a=-4.9$
$b=15$
$c=60$

$t_1<0$ reject
$t_2 \approx 5.35$

c. [6 points] A daredevil skier, V, did a stunt jump with a jetpack on their back. Suppose that the quadratic function $B(t)$ gives V's height (in meters) above their landing spot t seconds after going airborne. From the reports, it was gathered that V's jump lasted 8 seconds, that they jumped from a height of 48 meters above ground, and that they reached maximum height 3 seconds after they went airborne. Find a formula for $B(t)$.

① $B(8)=0$ ← because at $t=8$ they land
② $B(t) = a(t-3)^2+k$ ← because they reach the max height at 3 s.
③ $B(0)=48$

Subst. ① into ②

$B(8)=0 \Rightarrow a(8-3)^2+k=0$

$\Rightarrow a(25)+k=0 \rightarrow \boxed{k=-25a}$

Subst. ③ into ②

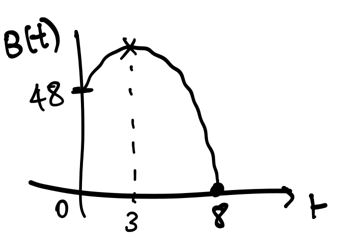
$B(0)=48 \Rightarrow a(0-3)^2+k=48$

$\rightarrow a(9)+k=48$

$9a-25a=48 \Rightarrow -16a=48$

$a=-3$ use $k=-25a$

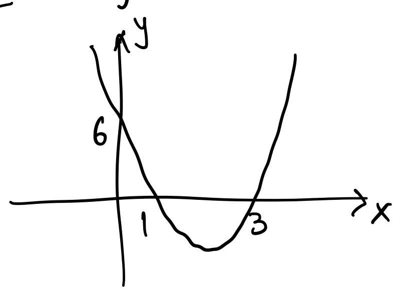
$k=-25(-3)=75$



$B(t) = -3(t-3)^2+75$

a k

Eg. Finding the formula from the zeros and the vertical intercept of a parabola.



Step 1 Write in factored form with zeros.

$y = a(x-1)(x-3)$ (*)

arbitrary constant

Step 2 Use the y-intercept

$x=0 \Rightarrow y=6$

Plug in into (*): $6 = a(0-1)(0-3)$

$\Rightarrow 6 = a(-1)(-3)$

$\Rightarrow 6 = 3a$

$\Rightarrow a=2$

Thus (*) becomes $\boxed{y=2(x-1)(x-3)}$

Standard form $y=ax^2+bx+c$

$$y = 2(x^2-3x-x+3)$$
$$= 2(x^2-4x+3)$$
$$= 2x^2-8x+6$$

Vertex form (Section 3.2).

Recall $y = ax^2+bx+c$ (standard form)

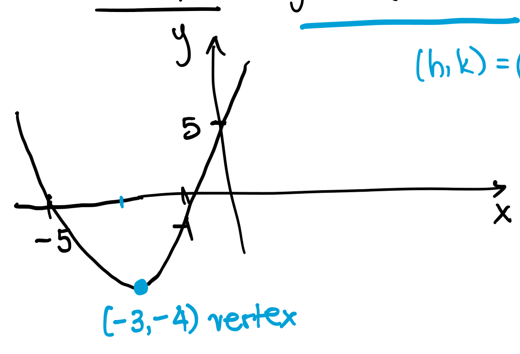
$y = a(x-r)(x-s)$ (factored form)

$\boxed{y = a(x-h)^2+k}$ (vertex form).

With the vertex given by the coordinate $\boxed{(h,k)}$ where h and k are both constants.

Example $g(x) = (x+3)^2-4$ vertex form.

$(h,k) = (-3,-4)$



Indicate the vertex on the graph.

So the vertex can be used to obtain the minimum of this function.

y-intercept: $x=0$

$$g(0) = (0+3)^2-4 = 3^2-4 = 9-4 = 5$$

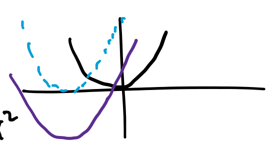
x-intercept: $y=0$ (zeros)

$$0 = (x+3)^2-4$$
$$4 = (x+3)^2$$
$$\pm 2 = x+3$$
$$x = -3 \pm 2 \begin{cases} -1 \\ -5 \end{cases}$$

When $x=-3$ what is $g(x)$? $g(-3) = (-3+3)^2-4 = 0^2-4 = -4$

$x<-3$, for example $x=-4$ $g(-4) = (-4+3)^2-4 = (-1)^2-4 = -3$

Basic parabola: $y=x^2$



$y = f(x) = x^2$

$g(x) = (x+3)^2-4$

$g(x) = f(x+3)-4$

horizontal shift to the left by 3
vertical shift downward by 4

Completing the square

This is used to convert from standard form to vertex form. $y=ax^2+bx+c \rightarrow y=a(x-h)^2+k$

Example 1 $y = x^2-6x+8$

$$= \left(x-\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2+8$$

always minus

$$= (x-3)^2-3^2+8$$
$$= (x-3)^2-9+8$$
$$= (x-3)^2-1$$

Check: $y = (x-3)(x-3)-1 = x^2-6x+9-1 = x^2-6x+8$ ✓

Example 2 $y = x^2+10x+4$

$$= \left(x+\frac{10}{2}\right)^2 - \left(\frac{10}{2}\right)^2+4$$

always minus

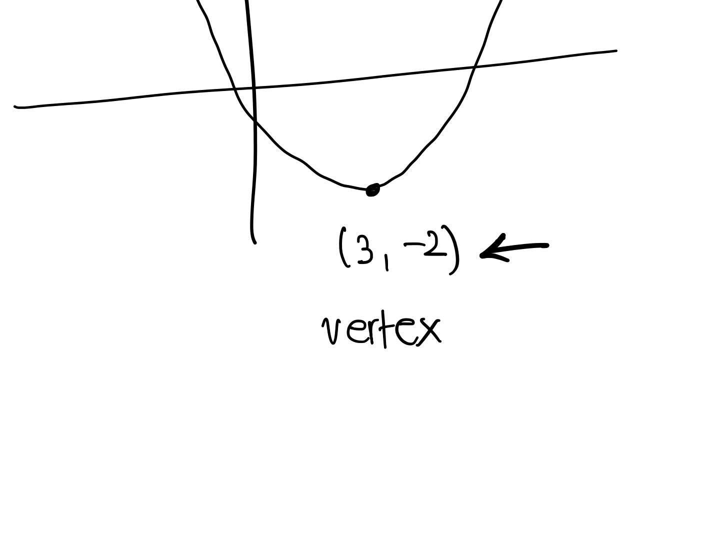
$$= (x+5)^2-5^2+4$$
$$= (x+5)^2-25+4$$
$$= (x+5)^2-21$$

check $y = (x+5)(x+5)-21$
$= x^2+10x+25-21$
$= x^2+10x+4$

Example 3 $y = -4x^2-12x-8$

$$= -4(x^2+3x+2)$$
$$= -4\left[\quad x^2+3x+2 \quad\right]$$
$$= -4\left[\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+2\right]$$
$$= -4\left[\left(x+\frac{3}{2}\right)^2-\frac{9}{4}+2\right]$$
$$= -4\left[\left(x+\frac{3}{2}\right)^2-\frac{1}{4}\right]$$
$$= -4\left(x+\frac{3}{2}\right)^2+1$$

Finding a formula given the vertex and another point on the parabola. $y=ax^2+bx+c$



(3, -2) ← vertex

Vertex form: $y = a(x-h)^2+k$

where (h,k) is the vertex

Using vertex: $y = a(x-3)^2-2$

To determine 'a' use the other point $(-3,4)$ (x y)

$$4 = a(-3-3)^2-2$$
$$4 = a(-6)^2-2$$
$$4 = 36a-2$$
$$6 = 36a$$
$$a = \frac{1}{6}$$

Substitute $a=\frac{1}{6}$ into $y=a(x-3)^2-2$ to get

$$\boxed{y = \frac{1}{6}(x-3)^2-2}$$