Power functions and proportionality (sec. 11.1)

Thursday, December 3, 2020 4:41 PM

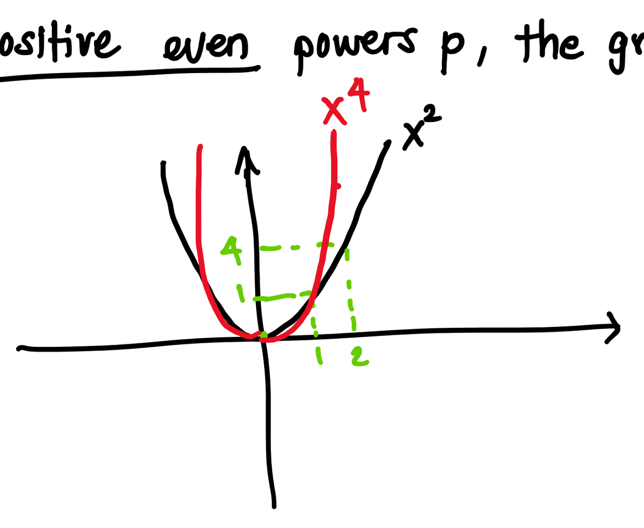
A **power function** is of the form $\boxed{f(x) = kx^p}$, where k and p are constants.

Note: In exponential functions the variable x is in the power, whereas in a power function it's in the base.

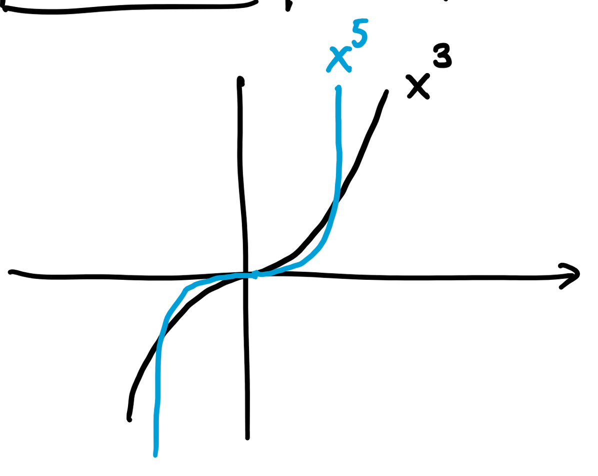
e.g. $f(x) = 2^x$ is exponential
$g(x) = 2(x)^3$ is a power function.

Powers p (when $k = 1$)

For positive even powers p, the graph is U-shaped.

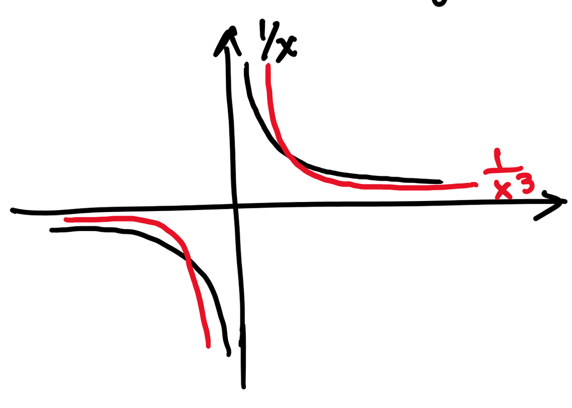


For positive odd powers p, the graph is "chair-shaped"

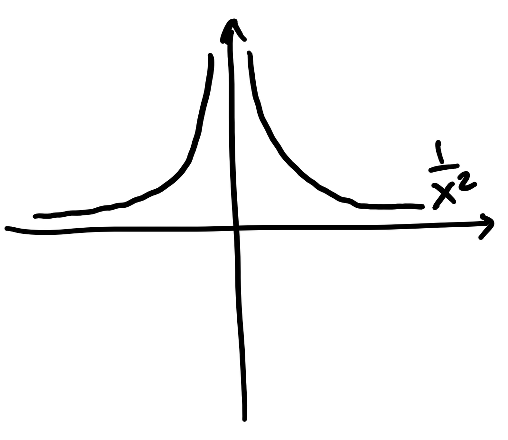


Let's now look at negative powers:

$\frac{1}{x} > \frac{1}{x^3}$ for large x



odd negative powers

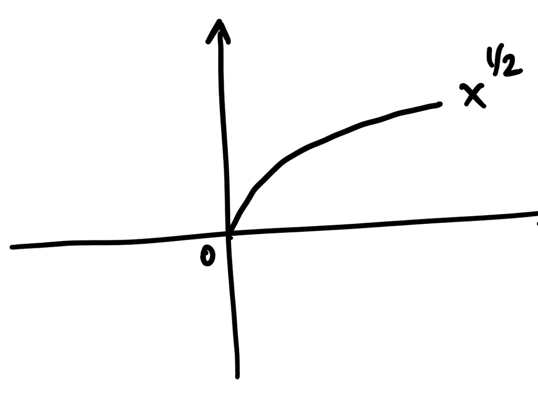


even negative powers

Power functions are functions of the form $y = kx^p$ where k & p are constants.

Consider positive fractional powers:

e.g. $y = x^{1/2}$



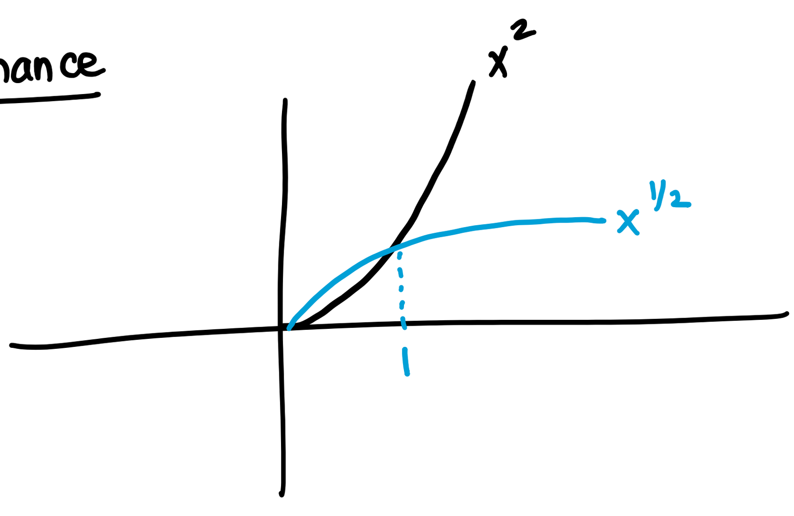
concave down

Any power function with $0<p<1$ has a graph that is concave down.
Any power function with $p>1$ has a graph that is concave up.

All power functions with $\boxed{p>0}$ have the same long-run behavior.
As $x \to \infty$ the value $x^p \to \infty$.

Dominance

e.g.



$x \geq 0$

In the long-run, as $x \to \infty$, x^2 grows faster that $x^{1/2}$ i.e. it's dominant.

→ If $0 < p < q$ the function $y = x^q$ dominates $y = x^p$.

Finding the formula for a power function

You do this by using two points on its graph.

Recall $\boxed{y = kx^p}$

e.g. Consider a power function with points $(2, 16)$ and $(3, 54)$. Find its formula:

$(2,16) \longrightarrow 16 = k(2)^p$
$(3,54) \longrightarrow 54 = k(3)^p$

We want to find the unknowns k and p:

$$\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$$

$$\frac{54}{16} = \frac{\cancel{k}(3)^p}{\cancel{k}(2)^p}$$

$$\frac{27}{8} = \left(\frac{3}{2}\right)^p \longrightarrow \ln\left(\frac{27}{8}\right) = p\ln\left(\frac{3}{2}\right)$$

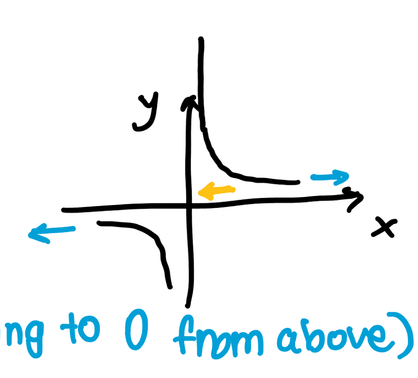
$$p = \frac{\ln\left(\frac{27}{8}\right)}{\ln\left(\frac{3}{2}\right)} = 3.$$

$\boxed{p=3}$

$16 = k(2)^3$
$16 = k(8)$
$\boxed{k=2}$

$y = kx^p \Rightarrow \boxed{y = 2x^3}$

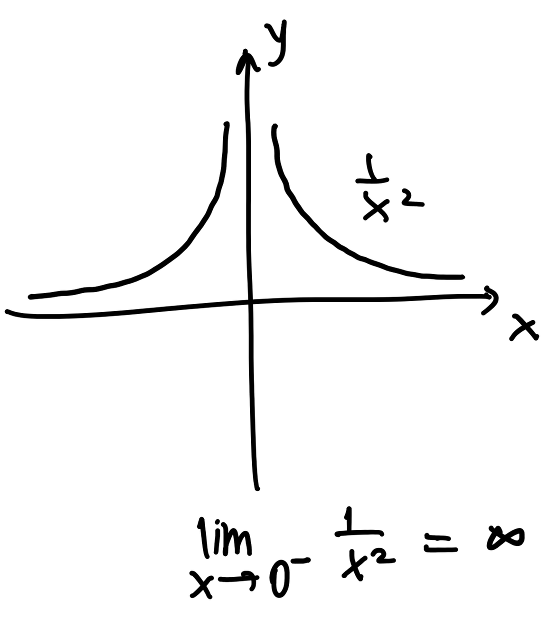
Asymptotes (limit notation)



Consider $\frac{1}{x}$ and $\frac{1}{x^2}$. As $x \to \infty$, $\frac{1}{x} \to 0^+$ (going to 0 from above)
As $x \to -\infty$, $\frac{1}{x} \to 0^-$ (going to 0 from below)

(limit notation: $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1}{x} = 0^+$

$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{1}{x} = 0^-$)



As $x \to 0^+$ (going to 0 from the right) $\frac{1}{x} \to +\infty$
As $x \to 0^-$ (going to 0 from the left) $\frac{1}{x} \to -\infty$

$\lim_{x\to 0^-} \frac{1}{x^2} = \infty$

Determine if the following are power functions and determine also k and p if it's a power function:

(1) $m(x) = 22(7^x)^2$ No (exponential)

(2) $h(y) = \frac{4}{\sqrt{16y}}$ YES $h(y) = \frac{4}{\sqrt{16}\sqrt{y}} = \frac{\cancel{4}}{\cancel{4}\sqrt{y}} = y^{-1/2}$ $\boxed{k=1,\ p=-1/2}$

(3) $f(x) = 4(x-1)^3$ YES shifted version of $4x^3$