Motion in space: velocity and acceleration

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Let $\vec{r}(t)$ be the position vector at time t then the velocity vector is given by

$$\vec{v}(t) = \lim_{h\to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h} = \vec{r}\,'(t)$$

The speed of a particle at time t is the magnitude of the velocity vector $|\vec{v}(t)|$

NB

$$\vec{v}(t) = \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle$$

$$|\vec{v}(t)| = |\vec{r}\,'(t)| = \text{rate of change of distance wrt time.}$$

$$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$

The acceleration of a particle is $\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t)$

EXAMPLE 3 A moving particle starts at an initial position $\mathbf{r}(0) = \langle 1, 0, 0 \rangle$ with initial velocity $\mathbf{v}(0) = \mathbf{i} - \mathbf{j} + \mathbf{k}$. Its acceleration is $\mathbf{a}(t) = 4t\,\mathbf{i} + 6t\,\mathbf{j} + \mathbf{k}$. Find its velocity and position at time t.

Since $\vec{a}(t) = \vec{v}\,'(t) \Rightarrow \vec{v}(t) = \int \vec{a}(t)\,dt = \int (4t\,\hat{i} + 6t\,\hat{j} + \hat{k})\,dt$

$$= 2t^2\,\hat{i} + 3t^2\,\hat{j} + t\hat{k} + \vec{C}$$

Using $\vec{v}(0) = \hat{i} - \hat{j} + \hat{k} = \vec{C}$

$$\Rightarrow \vec{v}(t) = (2t^2+1)\hat{i} + (3t^2-1)\hat{j} + (t+1)\hat{k}$$

Since $\vec{v}(t) = \vec{r}\,'(t) \Rightarrow \vec{r}(t) = \int \vec{v}(t)\,dt$

$$= \left(\tfrac{2}{3}t^3 + t\right)\hat{i} + (t^3 - t)\hat{j} + \left(\tfrac{1}{2}t^2 + t\right)\hat{k} + \vec{D}$$

Using $\vec{r}(0) = \langle 1, 0, 0 \rangle = \vec{D}$.

Example Let $x(t) = 10t$, $y(t) = 20t$, $z(t) = 30t - 5t^2$, $t \geq 0$

A toy is hit by a ball at the coordinate $(20, 40, 40)$
Is the ball moving upward or downward when it hits the toy?

$$(20, 40, 40) = (10t, 20t, 30t - 5t^2) \rightarrow t = 2.$$

$$\vec{v}(t) = \langle x'(t), y'(t), z'(t) \rangle = \langle 10, 20, 30 - 10t \rangle$$

At $t = 2$ $\vec{v}(2) = \langle 10, 20, 10 \rangle$

moving upward since $z'(2) > 0$.