# Logarithms and exponential models (sec 5.2)

Monday, November 2, 2020 5:56 PM

$y = ab^t$ and $y = ae^{kt}$ were the two exponential models we looked at.

How do you convert between the two?

$y = a(b)^t$ and $y = a(e^k)^t$ Note $(a^x)^y = a^{xy}$

$a(b)^t = a(e^k)^t$

$\Rightarrow$ $\boxed{b = e^k}$ or $\boxed{k = \ln(b)}$

using $\ln(b) = \ln(e^k)$

$\ln(b) = k$

Ex. Convert $Q = 7e^{0.3t}$ to the form $Q = ab^t$

$a = 7$

and $e^{0.3} = b$

$Q = 7(e^{0.3})^t$

$= 7(1.3499)^t$

Ex. Convert $P = 175(1.145)^t$ into $P = ae^{kt}$

$a = 175$

$b = 1.145 \Rightarrow k = \ln(b) = \ln(1.145) = 0.1354$

$\Rightarrow P = 175e^{0.1354t}$

## Doubling time and half life

Example (a) Find the time needed for the turtle population described by $P = 175(1.145)^t$, where $t$ is years, to double its initial size.

initial amount $t = 0$: $P = 175$

double of this amount is 350.

find $t$ for which $P = 350$ $\Rightarrow$ $175(1.145)^t = 350$

$(1.145)^t = \frac{350}{175} = 2$

$t \ln(1.145) = \ln(2)$

$\boxed{t = \frac{\ln(2)}{\ln(1.145)}}$ years.

b) How long does it take to quadruple its initial size?

**Half-life** If we start with an initial amount of $a$, what's the time it takes for it to be half of that?

e.g. $f(x) = a(3)^x$

$\frac{a}{2} = a(3)^x$

$\frac{1}{2} = 3^x$

$\ln\left(\frac{1}{2}\right) = x \ln(3)$

$\ln(1) = 0$

$x = \frac{\ln\left(\frac{1}{2}\right)}{\ln(3)} = \frac{\ln(1) - \ln(2)}{\ln(3)} = \frac{-\ln(2)}{\ln(3)}$