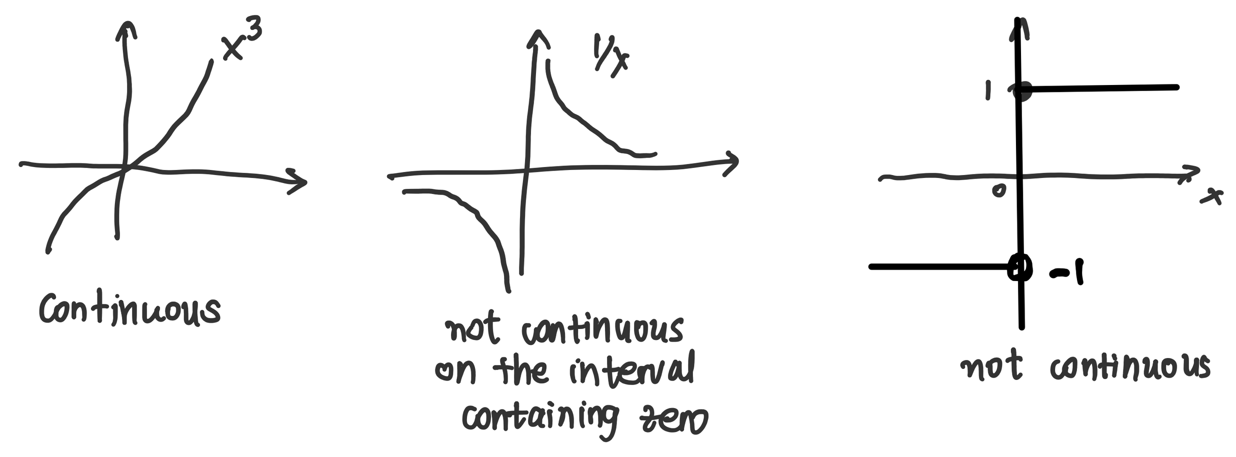
# Limits

Sunday, July 12, 2020 10:56 PM

## Intro to limits and continuity

A function is continuous on an interval if its graph has no jumps or holes in that interval.

$f(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$




Continuous

not continuous on the interval containing zero

not continuous

A function is continuous at a point if nearby values of the independent variable give nearby values of the function.

**Limit**

We write $\lim_{x \to c} f(x) = L$ if the values of $f(x)$ approach $L$ as $x$ approaches $c$.

**Limits of a continuous function**

If a function $f(x)$ is continuous at $x=c$, the limit is the value of $f(x)$ there

$$\boxed{\lim_{x \to c} f(x) = f(c)}$$

**Example** Use algebra to deduce what the limit is

$$\lim_{x \to 4} \frac{x^2-16}{x-4} = \lim_{x \to 4} \frac{\cancel{(x-4)}(x+4)}{\cancel{x-4}} = \lim_{x \to 4} x+4 = 8$$

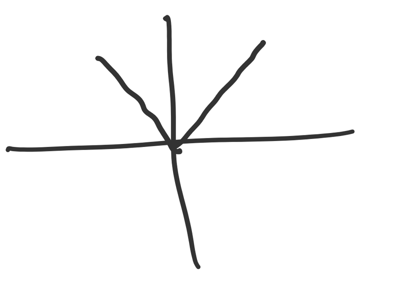
When does a limit not exist?

Eg $\lim_{x \to 2} \frac{|x-2|}{x-2} \Rightarrow \lim_{x \to 2^-} \frac{|x-2|}{x-2} = -1$

$\lim_{x \to 2^+} \frac{|x-2|}{x-2} = 1$

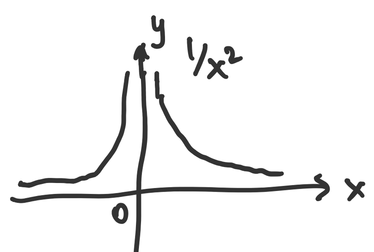
Left and right limits are different so the limit does not exist.

**Reminder** $y = |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$



$\frac{|x-2|}{x-2} = \begin{cases} \frac{-(x-2)}{x-2} = -1, & x < 2 \\ \frac{x-2}{x-2} = 1, & x > 2 \end{cases}$

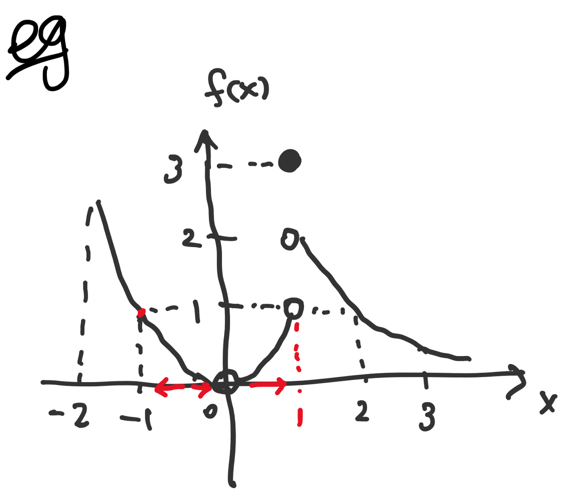
eg $\lim_{x \to 0} \frac{1}{x^2}$




DNE

because as $x \to 0$, $f(x) = \frac{1}{x^2}$ gets arbitrarily large and so it cannot approach a finite number L.

eg




a) $\lim_{x \to -1^+} f(x) = 1$

b) $\lim_{x \to 0^-} f(x) = 0$

c) $\lim_{x \to 0} f(x) = 0$

d) $\lim_{x \to 1^-} f(x) = 1$

e) $\lim_{x \to 1} f(x) = $ DNE

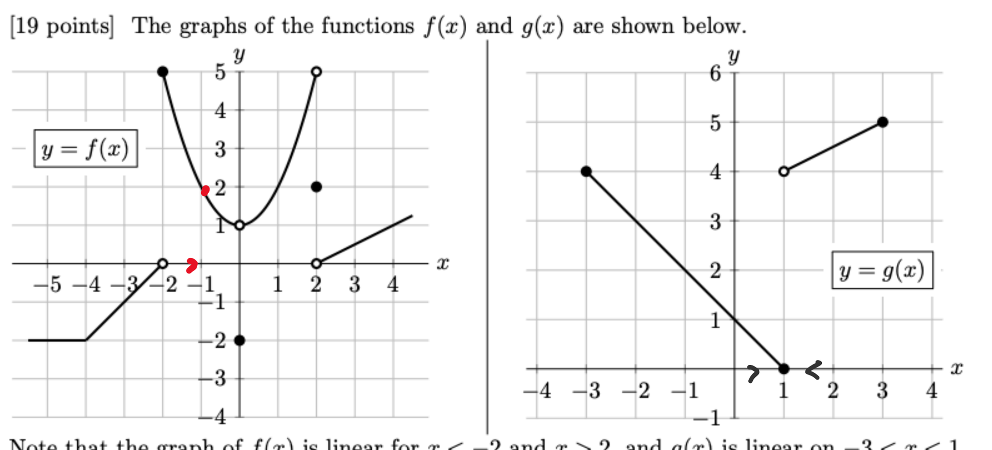
$\lim_{x \to 1^+} f(x) = 2$

**Theorem 1.2: Properties of Limits**

Assuming all the limits on the right-hand side exist:

1. If $b$ is a constant, then $\lim_{x \to c} (bf(x)) = b\left(\lim_{x \to c} f(x)\right)$.
2. $\lim_{x \to c} (f(x)+g(x)) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$.
3. $\lim_{x \to c} (f(x)g(x)) = \left(\lim_{x \to c} f(x)\right)\left(\lim_{x \to c} g(x)\right)$.
4. $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$, provided $\lim_{x \to c} g(x) \neq 0$.
5. For any constant $k$, $\lim_{x \to c} k = k$.
6. $\lim_{x \to c} x = c$.

**1.** [19 points] The graphs of the functions $f(x)$ and $g(x)$ are shown below.




Note that the graph of $f(x)$ is linear for $x < -2$ and $x > 2$, and $g(x)$ is linear on $-3 < x < 1$ and $1 < x < 3$.

For each of the following parts, find the given limit. If any of the quantities do not exist (including the case of limits that diverge to $\infty$ or $-\infty$), write DNE. If the limit cannot be found based on the information given, write NOT ENOUGH INFO. *You do not need to show any work.*

**a.** [2 points] Find $\lim_{x \to -1} f(x)$.

$\lim_{x \to -1} f(x) =$ 2

**b.** [2 points] Find $\lim_{t \to 2^-} 2(f(t)-3)$.

$\lim_{t \to 2^-} 2(f(t)-3) =$ 4

$\lim_{t \to 2^-} 2(f(t)-3)$
$= \lim_{t \to 2^-} 2f(t) - 6$
$= 2(5) - 6$
$= 4$

**c.** [2 points] Find $\lim_{x \to 1} f(x)g(x)$.

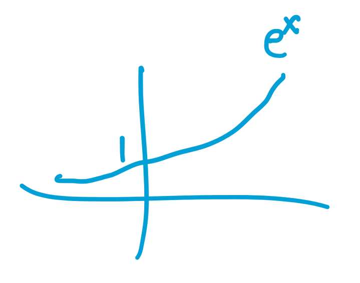
$\lim_{x \to 1} f(x)g(x) =$ DNE

c) $\lim_{x \to 1} f(x)g(x)$
$= \left(\lim_{x \to 1} f(x)\right)\left(\lim_{x \to 1} g(x)\right)$
$=$ DNE

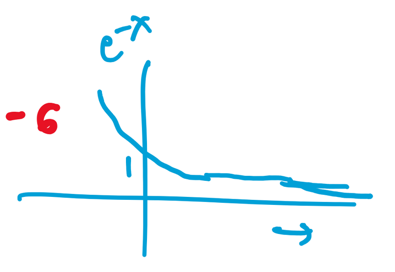
**d.** [2 points] Find $\lim_{x \to \infty} f(e^{-x})$.

$\lim_{x \to \infty} f(e^{-x}) =$ 1

$e^x$



$e^{-x}$



$\lim_{x \to \infty} e^{-x} = 0^+$

$\lim_{x \to \infty} f(e^{-x}) = \lim_{x \to 0^+} f(x)$
$= 1$

$g^{-1}(2) = -1$

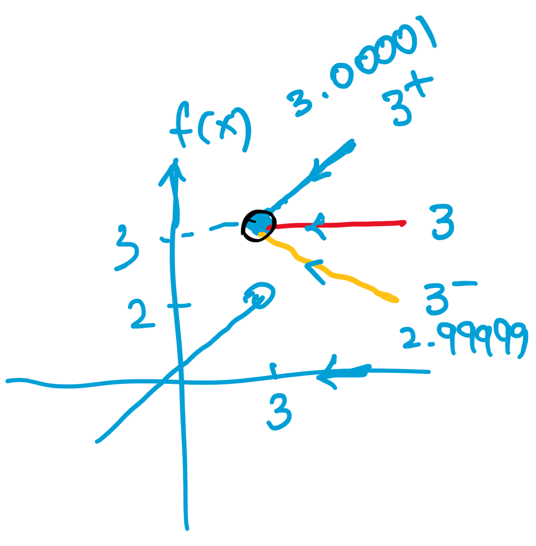
**e.** [2 points] Find $\lim_{x \to 2^+} g^{-1}(x)$.

$\lim_{x \to 2^+} g^{-1}(x) =$ -1

**f.** [2 points] Find $\lim_{h \to 0} \frac{f(3+h)-f(3)}{h}$.

$f'(3)$

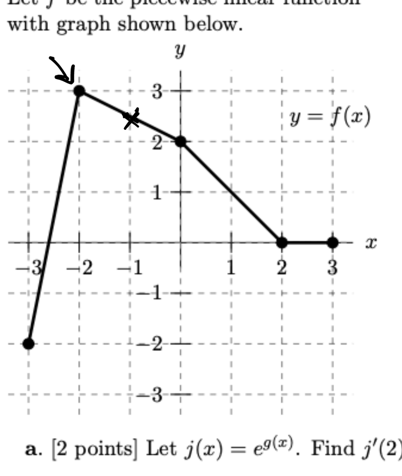
$\lim_{h \to 0} \frac{f(3+h)-f(3)}{h} =$ 1/2




$\lim_{x \to 3^+} f(x) =$

**2.** [12 points]

Let $f$ be the piecewise linear function with graph shown below.




The table below gives several values of a differentiable function $g$ and its derivative $g'$.
Assume that both $g(x)$ and $g'(x)$ are invertible.

| $x$ | $-2$ | $-1$ | $0$ | $2$ | $5$ |
|---|---|---|---|---|---|
| $g(x)$ | 21 | 11 | 5 | $-1$ | $-3$ |
| $g'(x)$ | $-12$ | $-8$ | $-4$ | $-2$ | $-0.4$ |

You are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown.

For each of parts **a.-f.** below, find the value of the given quantity. If there is not enough information provided to find the value, write "NOT ENOUGH INFO". If the value does not exist, write "DOES NOT EXIST".

**a.** [2 points] Let $j(x) = e^{g(x)}$. Find $j'(2)$.

$j'(x) = e^{g(x)} g'(x)$
$j'(2) = e^{g(2)} g'(2)$
$= e^{-1}(-2)$
$= -2/e$

**Answer:** -2/3

**b.** [2 points] Let $k(x) = f(x)f(x+2)$. Find $k'(-1)$.

$k'(x) = f'(x)f(x+2) + f(x)f'(x+2)$, $k'(-1) = f'(-1)f(1) + f(-1)f'(1)$
$= -\frac{1}{2}(1) + \frac{5}{2}(-1)$
$= -3$

$f(-1+2) = f(1)$

**Answer:**

**c.** [2 points] Let $h(x) = 3f(x) + g(x)$. Find $h'(-2)$.

$h'(x) = 3f'(x) + g'(x)$
$h'(-2) = 3f'(-2) + g'(-2)$

**Answer:** DNE

**d.** [2 points] Find $(g^{-1})'(2)$.

$g'(2.5)$

**Answer:** not enough info

**e.** [2 points] Let $m(x) = g(f(g(x)))$. Find $m'(2)$.

$m'(x) = g'(f(g(x))) \cdot f'(g(x)) \cdot g'(x)$

$m'(2) = g'(f(g(2))) f'(g(2)) \cdot g'(2)$

-1

**Answer:** not enough info

**f.** [2 points] Let $\ell(x) = \frac{f(x)}{g(2x)}$. Find $\ell'(-1)$.

quotient rule.

**Answer:** ≈ 0.1122