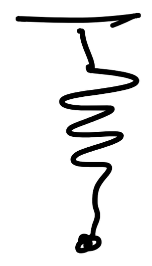
Intro to differential equations

Thursday, July 23, 2020 3:21 PM

A differential equation is an equation that states how a rate of change (a "differential") in one variable is related to the other variables.

eg The amount of stretch in the spring is directly related to the position of a particle, x. We can write this as a differential equation for the velocity

$$\frac{dv}{dt} = -kx \qquad \text{Hooke's law}$$



k spring constant.

eg Suppose we are interested in how fast an employee learns a new task. One theory claims that the more the employee already knows of the task, the slower he/she learns. In other words, if $y\%$ is the percentage of the task that the employee has already mastered and $\frac{dy}{dt}$ is the rate at which the employee learns then $\frac{dy}{dt}$ decreases as y increases

$$\boxed{\frac{dy}{dt} = 100 - y}$$

A formula for the solution

Let's suppose $y = 100 + Ce^{-t}$ is a solution. How do you check that?

$$LHS = \frac{dy}{dt} = -Ce^{-t}$$

$$RHS = 100 - y = \cancel{100} - (\cancel{100} + Ce^{-t}) = -Ce^{-t}$$

$\therefore$ LHS = RHS

The $y = 100 + Ce^{-t}$ satisfies the differential equation & must be a solution

a. [4 points] December is a busy time for cookie bakers and cookie eaters. Suppose that there is so much baking going on that cookies are added to the cookie supply of Ann Arbor at a rate of 10 pounds per minute. At the same time, 2% of the cookies are eaten every minute. Write a differential equation for the number of pounds C of cookies in Ann Arbor at time t, in minutes.

pounds of cookies per minute

$$\frac{dC}{dt} = 10 - 0.02C$$

pounds of cookies per minute.

pounds of cookies per minute

Example Wild rabbits were introduced to Australia in 1859. The behavior of the rabbit population P in Australia at a time t years after 1859 was modeled by the differential equation

$$P' = P + e^{-t} \qquad \leftarrow \frac{dP}{dt} = P + e^{-t}$$

Q For what value of B is

$$P = 3e^{t} + Be^{-t}$$

a solution to the differential equation?

S

$$LHS = \frac{dP}{dt} = 3e^{t} - Be^{-t}$$

$$RHS = P + e^{-t} = \underbrace{(3e^{t} + Be^{-t}) + e^{-t}}_{e^{-t}(B+1)} = 3e^{t} + (B+1)e^{-t}$$

Since it's a solution, we must have LHS = RHS

$$3e^{t} - Be^{-t} = 3e^{t} + (B+1)e^{-t}$$

Comparing coefficients

$$-B = B+1$$

$$2B = -1$$

$$\boxed{B = -\frac{1}{2}}$$