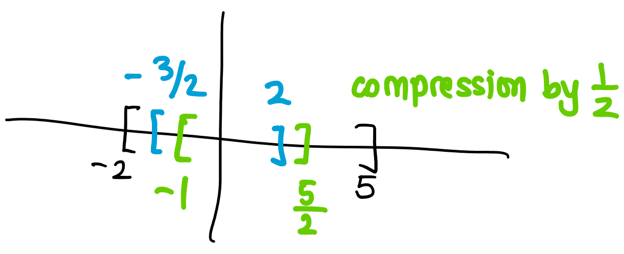
Horizontal stretches and combinations of transformations (sec 6.3)

Thursday, November 5, 2020 6:05 PM

1. [15 points]

a. [5 points] Suppose $f(x)$ is a function with domain $[-2, 5]$ and range $[7, 12]$. What are the domain and range of the transformation $g(x) = -f(2x+1) + 2$?



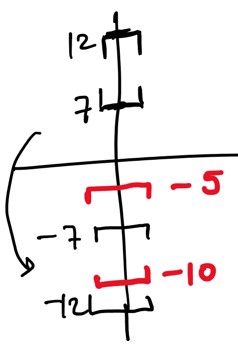
$g(x) = -f(2x+1)+2 = -f\left(2\left(x+\frac{1}{2}\right)\right)+2$

vertical changes • reflection in x-axis • shift up by 2

The domain of $g(x)$ is $[-3/2, 2]$.

The range of $g(x)$ is $[-10, -5]$.

range $[7, 12] \rightarrow [-12, -7] \rightarrow [-10, -5]$



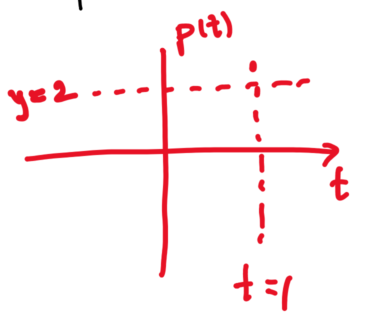
b. [4 points] Suppose $y = p(t)$ has vertical asymptote $t = 1$ and horizontal asymptote $y = 2$. Give the equations for a horizontal and vertical asymptote of the function $y = 2p(-t+3) + 1$. $= 2p(-(t-3))+1$

$y = 2 \rightarrow y = 4 \rightarrow y = 5$

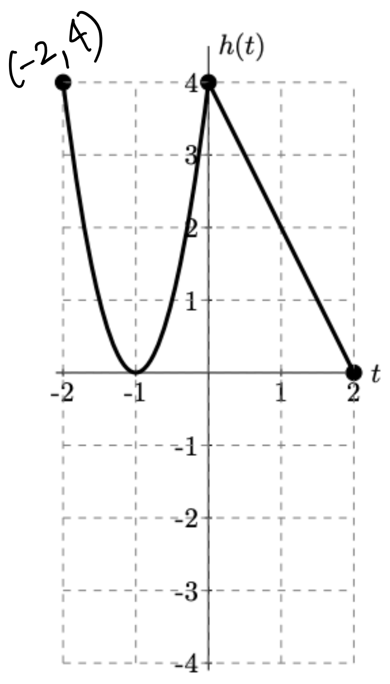
A horizontal asymptote of $2p(-t+3)+1$ is $y = 5$.

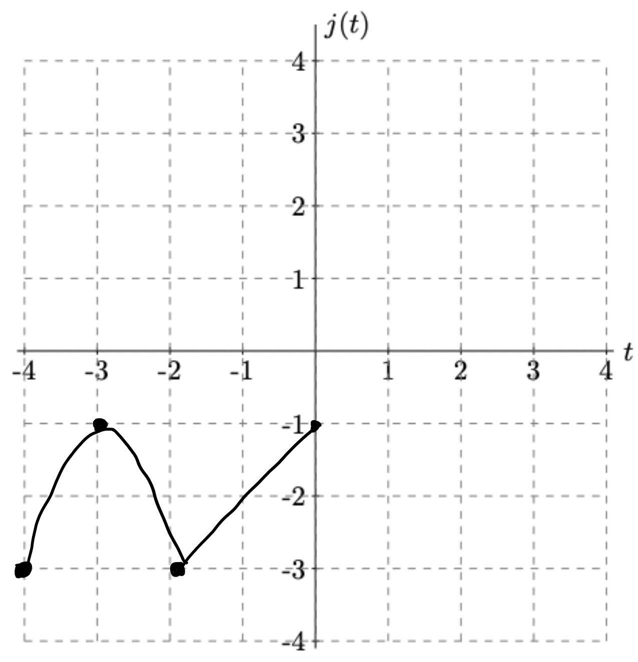
A vertical asymptote of $2p(-t+3)+1$ is $t = 2$.

$t = 1 \rightarrow t = -1 \rightarrow t = -1+3 = 2$



c. [6 points] A graph of the function $h(t)$ is given below. On the empty set of axes, carefully sketch a well-labeled graph of $j(t) = -\frac{1}{2}h(t+2) - 1$.





vertical.

$(-2,4) \rightarrow (-2,-4)$

$\rightarrow (-2,-2)$

$\rightarrow (-2,-3)$

hor.

$\rightarrow (-4,-3)$

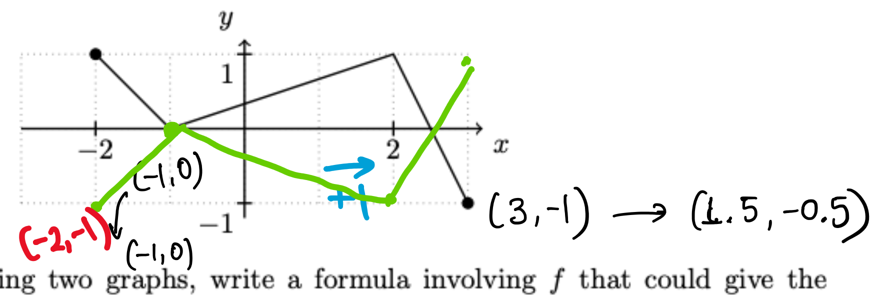
$(-1,0) \rightarrow (-3,-1)$

$(0,4) \rightarrow (-2,-3)$

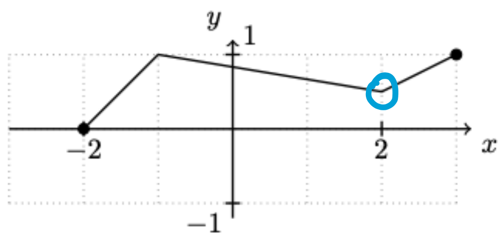
$(2,0) \rightarrow (0,-1)$

2. [12 points] Parts **a.** and **b.** of this problem are **unrelated** to each other.

a. [6 points] The graph of $y = f(x)$, defined on $[-2, 3]$ is given below.

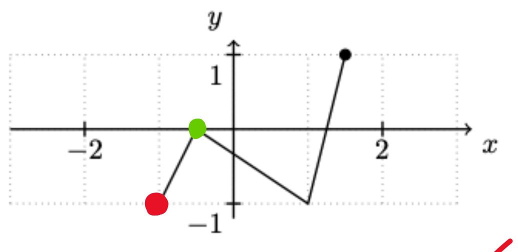


For each of the following two graphs, write a formula involving f that could give the graph.



This is the graph of

$y = \frac{1}{2}f(-(x-1)) + \frac{1}{2}$.



This is the graph of

$y = -f(2x)$.

b. [6 points] If a function $f(x)$ has domain $[0, 3)$, range $[-1, \infty)$, and a vertical asymptote at $x = 3$, find the domain, range and vertical asymptote of the function

$$g(x) = \frac{1}{3}f(-x+1) - 2. = \frac{1}{3}f(-(x-1)) - 2$$

(i) The domain of $g(x)$ is $(-2, 1]$.

$[0, 3) \rightarrow (-3, 0]$

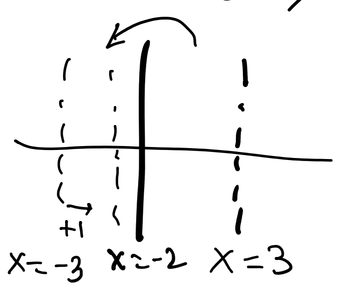
$\rightarrow (-2, 1]$

(ii) The range of $g(x)$ is $[-\frac{7}{3}, \infty)$.

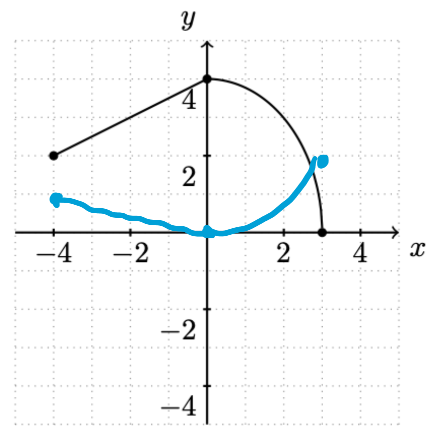
$[-1, \infty) \rightarrow [-\frac{1}{3}, \infty)$

$\rightarrow [-\frac{7}{3}, \infty)$

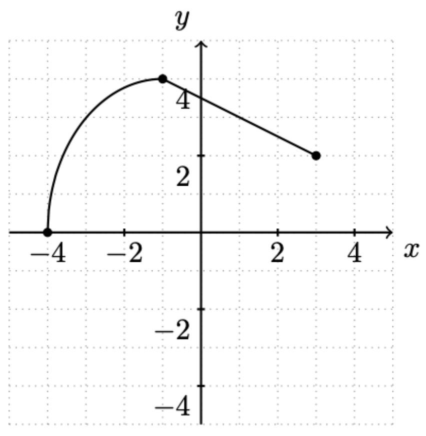
(iii) The vertical asymptote of $g(x)$ is $x = -2$.



7. [8 points] Consider the following graph of a function $y = q(x)$ defined on $[-4, 3]$.



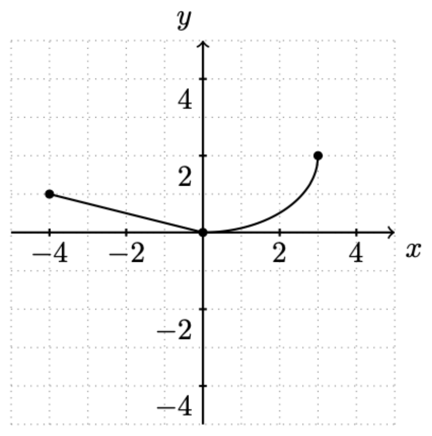
For each of the following graphs, if the graph is not a combination of shifts, stretches, compressions and reflections of the graph of $y = q(x)$, write NOT A TRANSFORMATION. Otherwise, write a formula for the function corresponding to graph in terms of $q(x)$.



This is the graph of

$y = q(-(x+1))$.

factored form



This is the graph of

$y = -\frac{1}{2}q(x) + 2$

$q(-x+1)$ wrong