Equilibrium solutions and their stability

Tuesday, July 28, 2020 8:56 PM

An equilibrium solution is constant for all values of the independent variable. (The graph is a horizontal line).

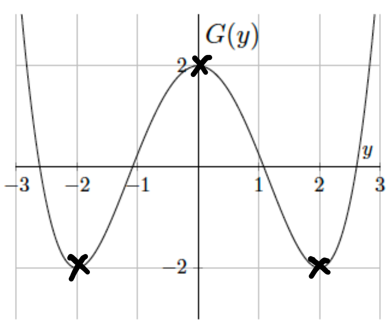
★ Equilibrium solutions are found using $\frac{dy}{dx}=0$ and solving for y. ★

Stability

An eqm sol^n is stable if a small change in the initial conditions gives a solution that tends towards the equilibrium as the independent variable goes to ∞.

An eqm sol^n is unstable if a small change in the initial conditions gives a solution that tends away from the equilibrium as the independent variable goes to ∞.

[11 points] The graph of $G(y)$ is shown below. Suppose that $G'(y) = g(y)$. Consider the differential equation $\frac{dy}{dt} = g(y)$.



derivative = 0

← y-axis

Eqm sol^n $\frac{dy}{dt}=0$

then solve for y

$\frac{dy}{dt}=g(y)=0$

$G'(y)=g(y)$

Note again that $\frac{dy}{dt} = g(y)$ and the given graph depicts $G(y)$ **not** $g(y)$.

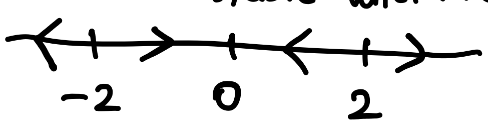
a. [6 points] The differential equation has 3 equilibrium solutions. Find the 3 solutions and indicate whether they are stable or unstable by circling the correct answer.

Equilibrium solution 1: -2 Stable Unstable

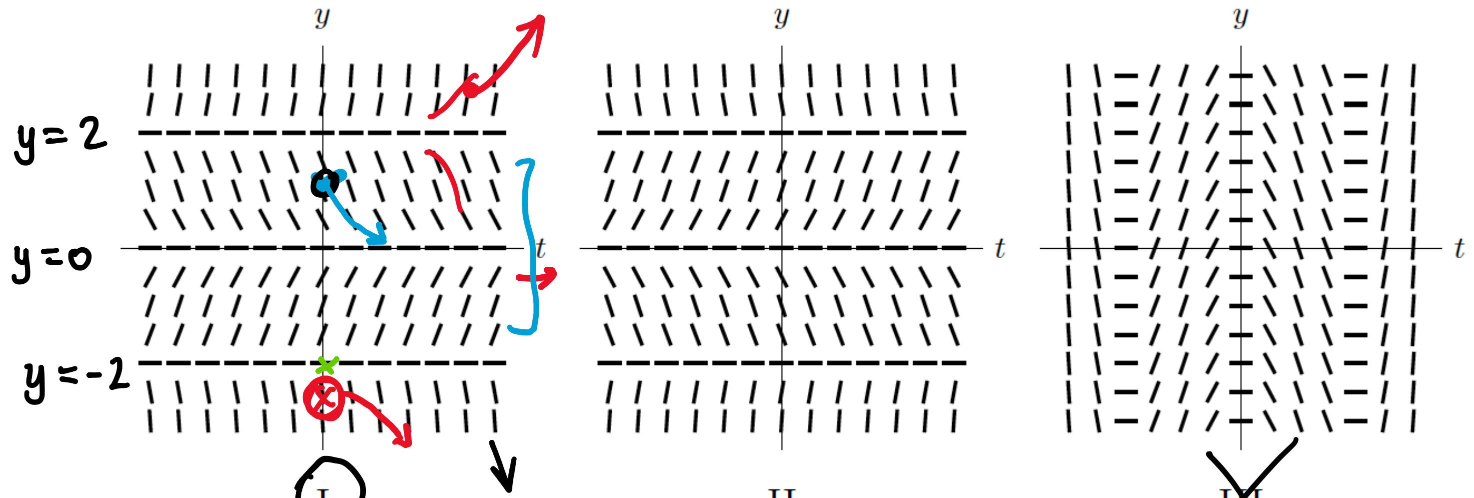
Equilibrium solution 2: 0 Stable Unstable

Equilibrium solution 3: 2 Stable Unstable

Example



b. [2 points] Circle the graph that could be the slope field of the above differential equation.



I II III

c. [3 points] Suppose $y_1(t), y_2(t)$ and $y_3(t)$ are all solutions of the differential equation with different initial conditions as indicated below:

- $y_1(t)$ solves the differential equation with initial condition $y(0) = -2$.
- $y_2(t)$ solves the differential equation with initial condition $y(0) = 1.5$.
- $y_3(t)$ solves the differential equation with initial condition $y(0) = -2.1$.

$y(t)$

$y(0)=-2$

At $t=0$ $y=-2$

Compute the following limits:

$\lim_{t\to\infty} y_1(t) = -2$ $\lim_{t\to\infty} y_2(t) = 0$ $\lim_{t\to\infty} y_3(t) = -\infty$

Example a) Consider a differential equation

$$\frac{dy}{dx} = (x-y)(y-2)$$

What are the equilibrium solutions? $\boxed{y=2}$

b) Use inequalities to describe the regions in the slope field where the solution curves are increasing.

$\frac{dy}{dx}>0$ $x-y>0$ and $y-2>0$ $\Rightarrow$ $y<x$ and $y>2$.

OR $x-y<0$ and $y-2<0$ etc.