Comparing exponential and linear functions (sec 4.2)

Thursday, October 29, 2020 6:08 PM

Identifying linear and exponential functions from a table:

x	20	25	30	35	40
f(x)	30	45	60	75	90
g(x)	1000	1200	1440	1728	2073.6

$m_1 = \frac{45-30}{25-20} = \frac{15}{5} = 3,\quad m_2 = \frac{60-45}{30-25} = \frac{15}{5} = 3, \ldots$

$\Rightarrow$ linear function

Point-slope formula

$y - 30 = 3(x-20)$

$y = 3x - 60 + 30$

$f(x) = y = 3x - 30$

Computing rates of change for g(x):

$m_1 = \frac{1200-1000}{25-20} = \frac{200}{5} = 40$

$m_2 = \frac{1440-1200}{30-25} = \frac{240}{5} = 48$

$m_1 \neq m_2$ not linear

Determining if it's exponential:

$\frac{1200}{1000} = 1.2, \quad \frac{1440}{1200} = 1.2, \quad \frac{1728}{1440} = 1.2$ etc.

So we see that each time x increases by 5, g(x) increases by a factor of 1.2.

These constant ratios imply g(x) is an exponential function.

★ Finding the formula for an exponential function: $y = ab^x$

x	20	25	30	35	40
g(x)	1000	1200	1440	1728	2073.6

Step 1: Plug in two points into $y = ab^x$ to get two equations

① $1000 = ab^{20}$

② $1200 = ab^{25}$ $\leftarrow$

Step 2: Divide one by the other

$\frac{\cancel{a}b^{25}}{\cancel{a}b^{20}} = \frac{1200}{1000} = 1.2$

$\frac{b^{25}}{b^{20}} = b^{25-20} = b^5$

$b^5 = 1.2$

$\boxed{b = (1.2)^{1/5}}$

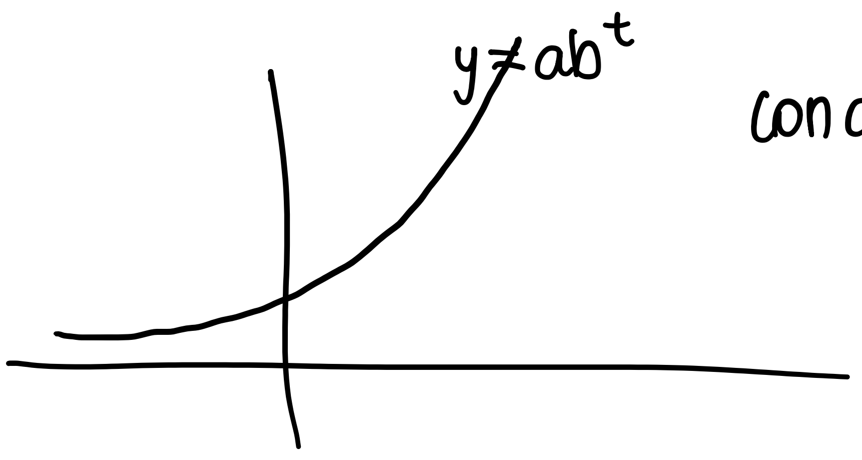
Step 3 Plug in b you solved in step 2 into either eqn ① or ② in step 1.

$b = (1.2)^{1/5}$ into ① $\Rightarrow$ $1000 = a\left((1.2)^{1/5}\right)^{20} = a(1.2)^4$

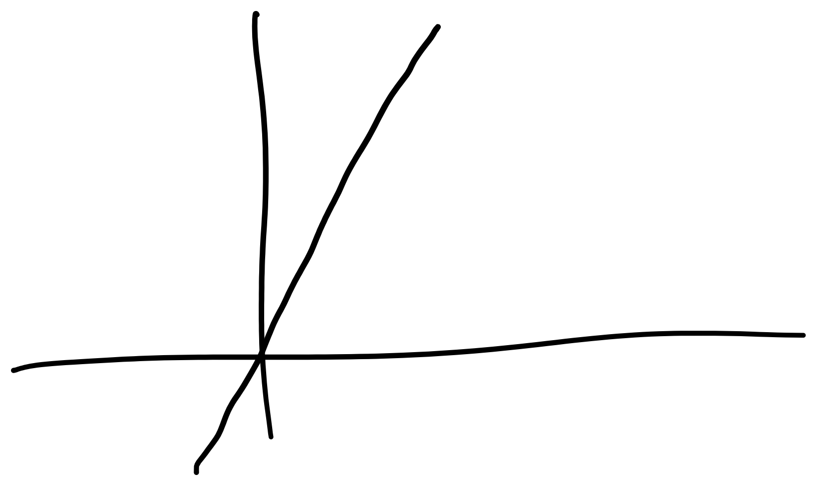
$(a^x)^y = a^{xy}$ $\Rightarrow$ $\boxed{a = \frac{1000}{(1.2)^4}}$

Step 4 Substitute into $y = ab^x$: $y = \frac{1000}{(1.2)^4}\left((1.2)^{1/5}\right)^x = \frac{1000}{(1.2)^4}(1.2)^{x/5}$

Note: Exponential growth always outpaces linear growth in the long run.



concave up



This is because the rate of change of the exponential function is increasing while the rate of change of a linear function is constant.