Chapter 1

Sunday, October 18, 2020 3:17 PM

Average rate of change $Q = f(t)$ for $a \le t \le b$

$$\frac{\Delta Q}{\Delta t} = \frac{Q(b) - Q(a)}{b - a}$$

Increasing functions have a positive rate of change

Decreasing functions have a negative rate of change.

A linear function has y values that change at a constant rate with the values of x.

$y = mx + b$ (m: slope, b: y-intercept.)

$y - y_1 = m(x - x_1)$ point-slope formula

In the line: $y = b + mx$

- The slope, m, is the change in y corresponding to an increase of 1 unit in x.
- The y-intercept, b, is the value of y when x=0. (b is measured in units of y).

Let $f(x) = 9 - x^2$.

a) Compute each of the following expressions and interpret each as an average rate of change:

(i) $\frac{f(1)-f(0)}{1-0}$ = -1

(ii) $\frac{f(3)-f(1)}{3-1}$ = -4

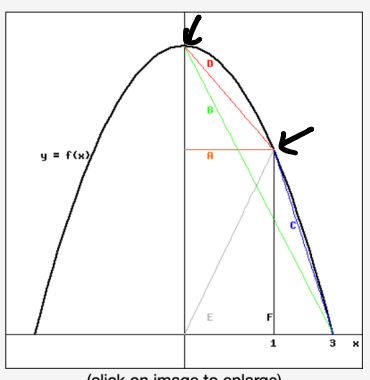
(iii) $\frac{f(3)-f(0)}{3-0}$ =

$f(1) = 9 - (1)^2 = 8$

$f(0) = 9 - 0 = 9$

$\frac{f(1)-f(0)}{1-0} = \frac{8-9}{1} = -1$

b) Based on the graph sketched below, match each of your answers in (i) - (iii) with one of the lines labeled A - F. Type the corresponding letter of the line segment next to the appropriate formula. Clearly not all letters will be used.



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D	$\frac{f(1)-f(0)}{1-0}$
	$\frac{f(3)-f(1)}{3-1}$
	$\frac{f(3)-f(0)}{3-0}$

$f(3) = 9 - (3)^2 = 0$

$f(1) = 9 - 1 = 8$

$\frac{f(3) - f(1)}{3-1} = \frac{0-8}{2} = -4$

You want to choose one long distance telephone company from the following options:

Company A charges $0.32 dollars per minute (no fixed monthly charge).
Company B charges $14.05 per month plus $0.24 per minute.
Company C charges a fixed rate of $50 per month.

Let $A(x)$, $B(x)$, and $C(x)$ denote the monthly charges of Company A, B, and C respectively for spending x minutes on long distance calls.

(a) Find a formula for the monthly cost of using Company A, $A(x)$:

$A(x)$ = 0.32x

(b) Find a formula for the monthly cost of using Company B, $B(x)$:

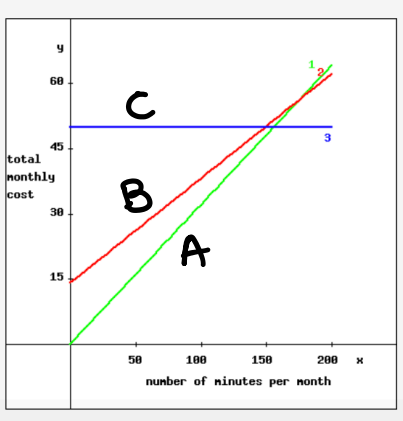
$B(x)$ = 0.24x + 14.05

(c) Find a formula for the monthly cost of using Company C, $C(x)$:

$C(x)$ = 50

(d) If you spend 125 minutes talking long distance in a month, which of the three companies will be cheapest? Enter the letter (A, B, or C) here: A

(e) Given the graph below, match the number of each graph to corresponding company it represents:



$B(0) = 14.05$

$A(125) = 40$

$B(125) = 44.05$

$C(125) = 50$

8. [11 points] The only food available on an unfortunate spaceship is dehydrated kale-spirulina tofu scramble (or "scramble") and dehydrated guacamole. These dehydrated foods must be rehydrated before consumption. A fixed-size portion of scramble with g grams of guacamole added to it needs $W(g)$ milliliters (ml) of water to be rehydrated. An astronaut reads a manual which specifies the following data:

g	11		36
$W(g)$	42	68	127

One value in the manual is missing; thankfully, the astronaut recalls that $W(g)$ is a linear function of g.

a. [3 points] Find an equation for $W(g)$. **Show your work.**

Const. rate of change.

$m = \frac{W(36) - W(11)}{36 - 11} = \frac{127 - 42}{25} = \frac{85}{25} = \frac{17}{5}$

$y - y_1 = m(x - x_1)$ $(x_1, y_1) = (11, 42)$

$W - 42 = \frac{17}{5}(g - 11)$

$W = \frac{17}{5}(g - 11) + 42$

$W(g)$ = _____

b. [2 points] Find the missing value in the table. Give your answer in **exact form**.

$W = 68$ solve for g

$\frac{17}{5}(g-11) + 42 = 68 \Rightarrow \frac{17}{5}(g-11) = 26 \Rightarrow g - 11 = \frac{130}{17}$

$g = \frac{130}{17} + 11 = \frac{317}{17}$

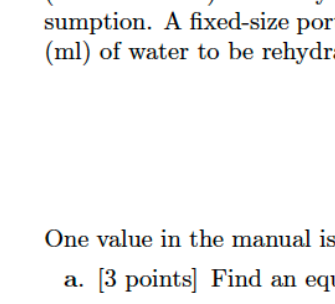
The missing value in the table is $g = 317/17$

c. [3 points] Give a practical interpretation for the vertical intercept of $W(g)$ in the context of the problem. Be sure to include units.

The amount of water in milliliters required to rehydrate the scramble itself when no guacamole is added.

d. [3 points] The graph of $S(g)$ is perpendicular to the graph of $W(g)$, and they have the same vertical intercept. Find an equation for $S(g)$. **Show your work.**

parallel



perpendicular

$W = \frac{17}{5}(g-11) + 42 = \frac{17}{5}g - \frac{187}{5} + 42$

$= \frac{17}{5}g + 5$

$m = \frac{17}{5}$

slope of perpendicular $-\frac{1}{m} = -\frac{5}{17}$

$S(g)$ = _____

$S(g) = -\frac{5}{17}g + 5$