Hybrid-State Free Precession for Measuring Magnetic Resonance Relaxation Times in the Presence of B_0 Inhomogeneities

Vladimir A. Kobzar[1], Carlos Fernandez-Granda[1,2], Jakob Assländer[3,4]
[1] New York University Center for Data Science, New York, NY, USA
[2] Courant Institute of Mathematical Sciences, New York University, New York, NY, USA
[3] Center for Biomedical Imaging, Dept. of Radiology, New York University School of Medicine, New York, NY, USA
[4] Center for Advanced Imaging Innovation and Research, New York University School of Medicine, New York, NY, USA

I. Synopsis

Magnetic resonance fingerprinting is a methodology for the quantitative estimation of the relaxation times $T_{1,2}$. An important challenge is to make estimation robust to inhomogeneities of the main magnetic field B_0. Precession sequences with smoothly varying parameters, such as balanced hybrid-state free precession (bHSFP) sequence, can be optimized for $T_{1,2}$-encoding performance. Previously, magnetic field deviations were assumed to be determined by a separate experiment. Here we develop a numerically optimized bHSFP sequence that takes into account variations in B_0 with the aim of mitigating bias due to B_0 inhomogeneities. Our numerical results indicate that this approach yields accurate $T_{1,2}$ estimates when B_0 inhomogeneities are unknown.

II. Introduction

In magnetic resonance fingerprinting (MRF) [1], the relaxation times T_1 and T_2 are determined quantitatively by matching the evolution of magnetization signal to a precomputed dictionary of patterns or "fingerprints" of tissues. However, inhomogeneities of the magnetic fields corrupt these estimates, producing for example so-called banding artifacts in the case of sequences with balanced gradient moments [1], [2]. The purpose of this work is to develop a numerically optimized fingerprinting sequence based on the hybrid state framework [2], [3] that incorporates B_0 estimation and mitigates bias due to B_0 inhomogeneities. In particular, the approach succeeds in suppressing banding artifacts.

III. Theory

In reference [2], it was shown that slow flip angle variations lead to a so-called "hybrid state", which allows us to solve the Bloch equation analytically in spherical coordinates. In this state, the entire spin dynamics is captured by the radial component r of the magnetization, which is controlled by the polar angle ϑ. The polar angle ϑ can be approximated [2], [4] by:

$$\sin^2\vartheta = \frac{\sin^2\frac{\alpha}{2}}{\sin^2\frac{\phi}{2}\cdot\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}}$$

where α denotes the flip angle (except in the vicinity of so-called stop bands, which are given by $|\sin\phi| \ll 1$). The phase

$$\phi = \phi^{nom} + \gamma\Delta B_0 T_R$$

is composed of the phase increment of the radio-frequency (RF) pulses ϕ^{nom} and the phase accumulated during the time T_R between pulses due to inhomogeneities in the main magnetic field. The gyromagnetic ratio is denoted by γ.

Expressing the magnetization in terms of the control parameters and field inhomogeneities makes it possible to optimize the sequence with respect to the T_1 and T_2 parameters, even if ΔB_0 is unknown.

IV. Methods

The Cramer-Rao bound [5], [6] is a lower bound on the error of any unbiased estimator of a parameter of interest. It is, therefore, a useful proxy for the sensitivity of the data with respect to the parameter, which can be minimized to optimize the measurements [7], [8], [9]. Here we follow such an approach to search for an efficient balanced hybrid-state free precession (bHSFP) sequence with anti-periodic boundary conditions (defined by $r(0) = -r(T_C)$, where T_C is the duration of one cycle of the pulse sequence [2]). We use the relative Cramer-Rao bound ($rCRB$), normalized by the duration of the experiment, as figure of merit and assume that the signal depends on unknown ΔB_0 variation parameters (as well as the magnetization and the relaxation times). We optimize an α pattern using the Broyden-Fletcher-Goldfarb-Shanno algorithm. In the most general approach, one would optimize ϕ^{nom} along with the flip angle pattern. However, we found this approach to show poor convergence behavior. Instead, we implicitly enforce equivalent encoding at all ΔB_0 offsets (within the limits of the RF-hardware) by sweeping through $\gamma\Delta B_0 T_R \in [0, 2\pi]$ while repeating the same α pattern.

For consistency with hybrid state conditions [2], the changes in the flip angles in consecutive RF pulses $\Delta\alpha$ were constrained by

$$|\Delta\alpha| \leq \max\left\{\sin^2\frac{\alpha}{2} - \frac{5}{2}\left(1-\sqrt{E_2}\right)^2, 0\right\} \tag{1}$$

where $E_2 = \exp(-T_R/T_2)$.

V. Results and Discussion

The optimized bHSFP sequences exhibit smooth ϑ and α patterns (Figure 1), and the ϑ pattern is similar to one found when neglecting field inhomogeneities ($\Delta B_0 = 0$) [2], [3]. Also when plotting the optimized $rCRB$ as a function of T_C (Figure 2), we observe that the optimal duration $T_C = 3.8$ s is also comparable to the ones found in literature for the idealized case. Lastly, Figure 3 indicates that the sequences optimized for unknown B_0 have similar $rCRB$ as the sequences optimized for known B_0 variations. (Using the latter sequence in a setting when B_0 variations are unknown would have resulted, however, in a significantly higher $rCRB$.) Moreover, the horizontal lines disappear in Figure 3, which indicates that T_1 and T_2 can be estimated without banding artifacts.

VI. Conclusion and Outlook

Our results indicate that incorporating B_0 variations while designing bHSFP sequences for magnetic resonance fingerprinting is a promising avenue to achieve robust estimates of T_1 and T_2 in the presence of B_0 inhomogeneities. Future work will include validation on phantom and in vivo scans. In addition, we will extend our methodology to account for variations in the RF field B_1.

VII. Acknowledgements

V.A.K. is supported by the Moore-Sloan Data Science Environment at New York University. C.F. is supported by NSF award DMS-1616340. J.A. is supported by the research grants NIH/NIBIB R21 EB020096 and NIH/NIAMS R01 AR070297, and this work was performed under the rubric of the Center for Advanced Imaging Innovation and Research (CAI2R, www.cai2r.net), a NIBIB Biomedical Technology Resource Center (NIH P41 EB017183).

References

[1] Dan Ma, Vikas Gulani, Nicole Seiberlich, Kecheng Liu, Jeffrey L. Sunshine, Jeffrey L. Duerk, and Mark A. Griswold. Magnetic resonance fingerprinting. Nature, 495(7440):187–192, 2013.

[2] Jakob Assländer, Riccardo Lattanzi, Daniel K Sodickson, and Martijn A. Cloos. Hybrid state free precession in nuclear magnetic resonance. arXiv:1807.03424 [physics.med-ph], 2018.

[3] Jakob Assländer, Daniel K Sodickson, Riccardo Lattanzi, and Martijn A Cloos. Hybrid State Free Precession for Measuring Magnetic Resonance Relaxation Times. In Proc. Intl. Soc. Mag. Reson. Med, 2018.

[4] Ray Freeman and H. D W Hill. Phase and intensity anomalies in fourier transform NMR. J. Magn. Reson., 4(3):366–383, 1971.

[5] Calyampudi Radhakrishna Rao. Information and the Accuracy Attainable in the Estimation of Statistical Parameters. Bull. Calcutta Math. Soc., 37(3):81–91, 1945.

[6] Harald Cramér. Methods of mathematical statistics. Princeton University Press, Princeton, NJ, 1946.

[7] J.A. Jones, P. Hodgkinson, A.L. Barker, and P.J. Hore. Optimal Sampling Strategies for the Measurement of Spin–Spin Relaxation Times. J. Magn. Reson. Ser. B, 113(1):25–34, 1996.

[8] J. A. Jones. Optimal sampling strategies for the measurement of relaxation times in proteins. J. Magn. Reson., 126(126):283–286, 1997.

[9] Bo Zhao, Justin P Haldar, Kawin Setsompop, and Lawrence L Wald. Optimal Experiment Design for Magnetic Resonance Fingerprinting. In Eng. Med. Biol. Soc. (EMBC), IEEE 38th Annu. Int. Conf., number 1, pages 453–456, 2016.

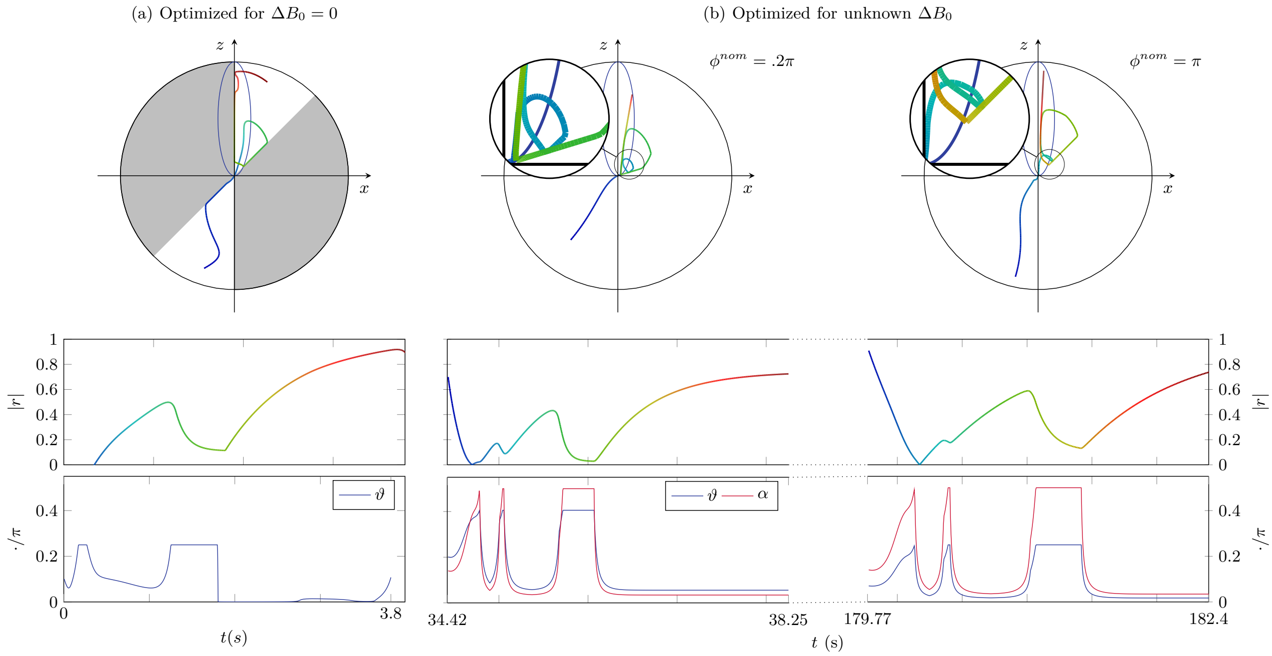

Fig. 1: The dynamics in (a) result from optimizing an α sequence of length $T_C = 3.8$ s where ΔB_0 is assumed to be zero (cf. Fig 4 in [2]). The polar angle was constrained to $0 \leq \vartheta \leq \pi/4$. The dynamics in (b) result from optimizing the same sequence concatenated 96 times under the assumption that ΔB_0 variations are unknown. In each period of length T_C, a different phase offset ϕ^{nom} is introduced, with ϕ^{nom} uniformly distributed between $-\pi$ and π. The flip angle was constrained to $0 \leq \alpha \leq \pi/2$, and the changes of α were limited in accordance with (1). The blue ellipse indicates the steady state of balanced steady-state free precession sequences.

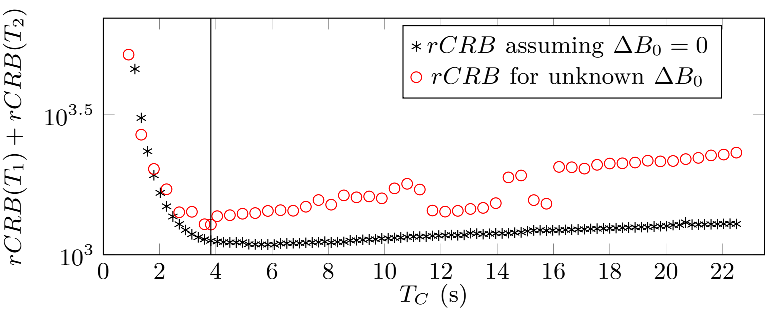

Fig. 2: The bHSFP trajectories described by the relaxation of the Bloch Equations are optimized for different lengths (T_C) of the flip angle α sequences. The rCRB values in the scenario where B_0 variations are unknown indicate that $T_C = 3.8$ s is optimal, as marked by the vertical bar. (These bounds result from a nonconvex optimization where the parameter space grows with T_C, and convergence issues become apparent for larger T_C.) The rCRB in the scenario where B_0 variations are known is also depicted and the optimal $T_C = 3.8$ s is consistent with prior work (cf. Fig. 3 in [2]).

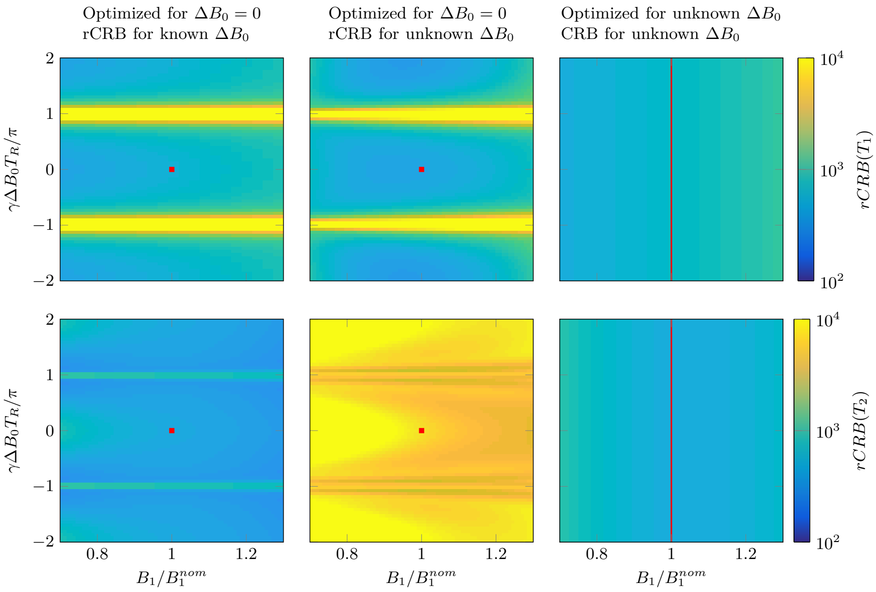

Fig. 3: The performance of the optimized experiments is illustrated through plots of the rCRB values, which provide a lower bound for the noise in the retrieved relaxation times. The performance is illustrated as a function of the magnetic field, which is parameterized by the main magnetic field variation ΔB_0, and by the RF field variation B_1. During the optimization, the nominal values of the inhomogeneities were fixed, as shown by the red squares / red lines. The experiments have $T_C = 3.8$ s (cf. Fig. 2). (The rCRB color bar is in log scale.)