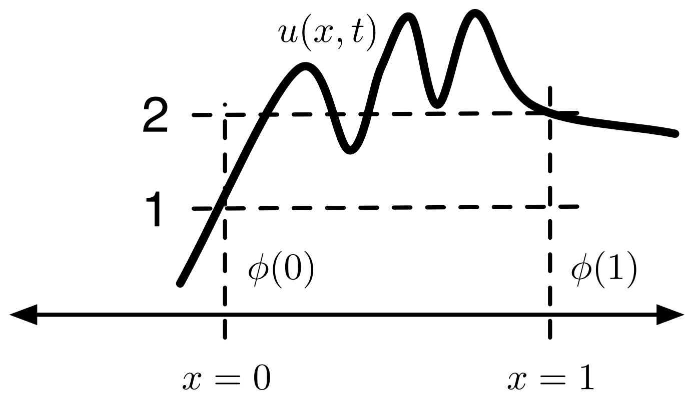
Name:______________________ Quiz Score:____/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the box to the left of the question. If there are any other issues, please ask the instructor.

1. Suppose at a fixed time t the concentration $u(x,t)$ of a chemical is given by the graph in the below figure.



Let $M(t) = \int_0^1 u(x,t)\mathrm{d}x$ be the total mass of the chemical in $[0,1]$.

Also suppose we somehow magically know $u_x(0,t) = 2$ and $u_x(1,t) = -1/2$.

4 (a) If the flux rule is $\phi = -\partial_x u + u$, determine whether the total amount of chemical $M(t)$ is growing or shrinking at any time.

2 (b) Determine the PDE associated with this flux rule.

Solution:

(a) Here, the big thing to remember is the continuity law:

$$\partial_t u = -\partial_x \phi + \underbrace{R(x)}_{0},$$

Integrating this from $x = 0$ to $x = 1$ and using the definition of $M(t)$ yields our other standard continuity equation

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \phi(0) - \phi(1).$$

This is basically all we need to know for this problem. Specifically, at the left end, we can plug in the values using the given flux rule:

$$\phi(0) = -2 + 1 = -1$$

and

$$\phi(1) = -(-1/2) + 2 = 5/2$$

so plugging these back into the continuity equation we get

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -1 - 5/2 = -7/2.$$

Because this is negative, it says the growth of total amount of stuff is negative, so it is going *down (shrinking)*.

rubric: 2 pts for continuity equation, 1 pt for getting the right $\frac{\mathrm{d}M}{\mathrm{d}t}$, 1 pt for right interpretation.

(b) This problem wasn't meant to be a trick but seemed to trip a lot of people Up. Basically, we just take our continuity equation and plug in the flux rule to get a PDE just for u:

$$\partial_t u = -\partial_x \phi = -\partial_x \{-\partial_x u + u\} = \partial_{xx} u - \partial_x u.$$

So, we could write this as a concise PDE

$$\partial_t u = \partial_{xx} u - \partial_x u.$$

rubric: 1 pt for using the continuity equation, 1 pt for correct PDE.

2 2. Why do we need partial differential equations (PDEs) to model? Why aren't ODE (ordinary differential equations) sufficient? What do they describe differently?

Solution: A wide array of answers here are totally acceptable. Basically, ODEs describe how a single quantity changes in time *or* space. However, PDEs describe how things change in time *and* space. That is, things are changing in more than 1 dimension. You could say this often comes from things moving, diffusing, or from some other flux rule.

rubric: 1 pt for any coherent thought, 1 pt for something about more than 1 dimension.

2 3. Fourier's law states that the flux $\phi(x)$ of heat (say, in a rod) described by temperature $T(x)$ obeys the relation

$$\phi(x) = -k\partial_x T(x).$$

Explain what this means in words and why it maybe makes some physical sense.

Solution: We discussed this in class: basically it says that heat flows *down* gradients of temperature. That is, heat flows from high heat to low heat. This makes physical sense: if we heat one side of a bar, the heat at that end will eventually flux over to the other side. When does it stop? When the temperature is equal in the bar! Here, k is the thermal conductivity. Some materials flow heat faster than others and k determines this.

rubric: 1 pt for any coherent thought, 1 pt for something about heat flowing down gradients or from high temperature to low temperature.