Math 3150 – Review checklist

I don't claim that this is a completely comprehensive list of topics on the final but it covers the majority of the topics that could possibly show up.

Chapter 1: Introduction to PDE's for Transport

- 1.1
  - The notion of flux/net flux
  - The notion of a source/net source
  - What does it mean for a quantity to be conserved?
  - The Conservation Principle: Net rate of change = net flux+net source
  - What does it mean for a quantity to be at equilibrium?
- 1.2
  - The continuity equation in 1D: $\frac{\partial u(x,t)}{\partial t} = -\frac{\partial \phi(x,t)}{\partial x} + R(x,t)$
  - The continuity equation in 2,3D
  - How the continuity equation is derived
  - How to use the continuity equation to find relationships between $u(x,t)$, $\phi(x,t)$, and $R(x,t)$.

Chapter 2: Mathematical Tools for PDE's

- 2.1
  - Linear Algebra review
  - Vector spaces, linear independence, span, basis
  - Function spaces
- 2.2
  - Be able to solve first and second order homogeneous ODE's (with or without boundary conditions) with constant coefficients using separation of variables or the characteristic equation.
- 2.3
  - Linear differential operators
  - Eigenvalue/function problem for linear differential operators
  - Given a linear differential operator L and an eigenvalue λ, find the eigenfunction

- Given a linear differential operator L and boundary conditions, find the eigenvalues and eigenfunctions
- There will be **NO** questions about writing boundary conditions in the form $B_a u_a + B_b u_b = 0$

- 2.5
  - Inner products and inner product spaces
  - What are the inner products for $\mathbb{R}^n$ and function spaces?
  - What is a norm?
  - What does it mean for two vectors to be orthogonal?

- 2.6
  - What does it mean for a set of functions to be orthogonal or orthonormal?
  - Given a set of polynomials S on $[a, b]$ of degree $< n$, find a polynomial of degree n that is orthogonal to S.

- 2.7
  - Find the least squares approximation of a function using a set of orthogonal functions $\{\phi_1(x), \phi_2(x), \phi_3(x), ..., \phi_n(x)\}$
  - Understand why the least squares approximation of function $f(x)$ is the optimal approximation using a linear combination of basis functions
  - Use the formula for error of the least squares approximation. Understand what happens as $n \longrightarrow \infty$

Chapter 3: Fourier Series

- 3.1 and 3.2
  - Fourier series on $[-\pi, \pi]$. What are the orthogonal functions used?
  - What is periodic extension, piecewise continuous, piecewise smooth?
  - Calculate the Fourier series on $[-L, L]$ to approximate a function

- 3.3
  - Fourier sine and Fourier cosine series
  - Even extensions and an odd extensions
  - What is the relationship between Fourier sine and cosine series and even and odd extensions?

- 3.4

- Plotting and interpreting energy spectra **[not tested]**

Chapter 4: Solutions of Transport PDEs

- 4.1
  - Fourier's law
  - Definition 4.1, the definition of the general heat equation
  - The common boundary conditions, both mathematical formulation and the physical definitions
- 4.2
  - Fick's law
  - Defintion of the diffusion equation
  - The common boundary conditions with both their physical and mathematical discriptions
- 4.3
  - Be able to recognize an equilibrium problem
  - Find equilibrium solutions the the heat/diffusion equations with fixed temperature or fixed flux boundary conditions and possibly a relatively simple source term
- 4.4
  - Nothing
- 4.5
  - Know the steps in the separation of variables algorithm
  - Be able to perform each step in the algorithm with reasonable accuracy
  - I will provide a list of necessary integrals especially those involving integration by parts etc.
  - Know the behavior of the heat/diffusion equation with insulated/fixed temperature boundary conditions as $t \longrightarrow \infty$
  - know how to solve Laplace's equation $u_{xx} + u_{yy} = 0$ with 4 specified boundaries.
- 4.6
  - Be able to recognize a non-homogeneous solution
  - Be able to combine an equilibrium solution and homogeneous solution to find the solution to a non-homogeneous problem
  - Find the particular solution to a non-homogeneous problem

Chapter 5: PDEs for Oscillations and Waves

- 5.1
  - Understand the assumption of the wave equation
  - Definition 5.1, the wave equation, boundary conditions, and initial conditions
  - Be able to solve the wave equation with separation of variables
- 5.2
  - Find and analyze solutions to the wave equation using d'Alembert's formula
  - Sketch solutions to the wave equation