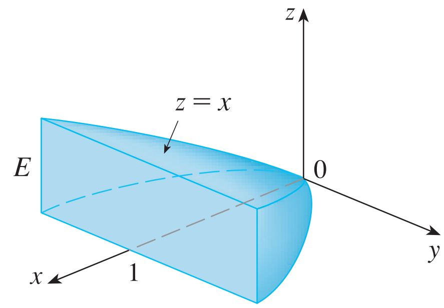
Name:____ Quiz Score:_____/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

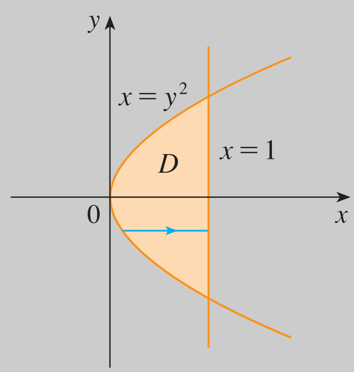
[6] 1. Set-up and evaluate the triple integral

$$\iiint_E 1 \, dV,$$

where E is the solid bounded by $x = y^2, x = z, z = 0$ and $x = 1$ shown to the right. *Hint:* try dz first.



Solution: If we first consider integrating over dz we must consider what the bounds of z are. We see the top of z is always the plane $z = x$ and the bottom is $z = 0$. Thus, those are our bounds for z. Next, we look from above (in the z direction) to determine the x, y bounds, which we can see below.



From this, we could set this up as a type 1 or type 2 integral, but specifying x first seems easiest, as it's clear it goes from $x = y^2$ to $x = 1$. From there, y goes from the intersection of these two curves, which is $y = -1$ and $y = 1$, so our final integral is

$$\int_{-1}^{1} \int_{y^2}^{1} \int_{0}^{x} 1 \, dz \, dx \, dy.$$

We can evaluate this inside out:

$$\begin{aligned}\int_{-1}^{1}\int_{y^2}^{1}\int_{0}^{x} 1\,dz\,dx\,dy &= \int_{-1}^{1}\int_{y^2}^{1} x\,dx\,dy\\ &= \int_{-1}^{1}\left[\frac{x^2}{2}\right]_{x=y^2}^{x=1} dy\\ &= \frac{1}{2}\int_{-1}^{1}(1-y^4)\,dy\\ &= \frac{1}{2}\left[y-\frac{y^5}{5}\right]_{y=-1}^{y=1} = \frac{4}{5}.\end{aligned}$$

4 2. Set up an integral for (but do not compute) the volume between the spheres $x^2+y^2+z^2=2z$ and the sphere $x^2+y^2+z^2=4$ chopped off to only include above $z=0$.

Solution: **There was a typo in the original problem that had $\rho=\sqrt{2}$, which messes up the setup of the problem, but not really the solution at all.** This whole problem effectively boils down to converting everything to spherical. Examining the first sphere's equation and converting to spherical, we get

$$\rho^2 = 2\rho\cos\phi \qquad\Longrightarrow\qquad \rho = 2\cos\phi.$$

The second (outside) sphere reduces to

$$\rho^2 = 4 \qquad\Longrightarrow\qquad \rho = 2.$$

Thus, these specify the ranges for ρ, but now we must decide ϕ,θ. Since we are only considering the top half of the sphere, ϕ ranges between $\phi=0$ and $\phi=\pi/2$. We have full rotational symmetry so $\theta\in[0,2\pi]$. Thus, the final integral ends up being, remembering the conversion factor $dV=\rho^2\sin\phi d\rho d\phi d\theta$,

$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_{2\cos\phi}^{2} 1\rho^2\sin\phi\,d\rho\,d\phi d\theta.$$