Name:_____ Quiz Score:_____/10

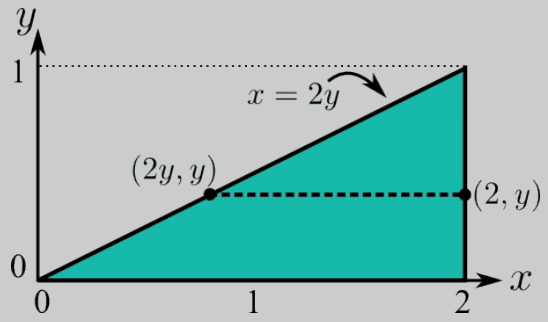
Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

3 1. Consider the integral

$$\iint_D f(x,y)\,dA = \int_0^1 \int_{2y}^2 xy^2\,dx\,dy.$$

Draw the region D which this integral corresponds to.

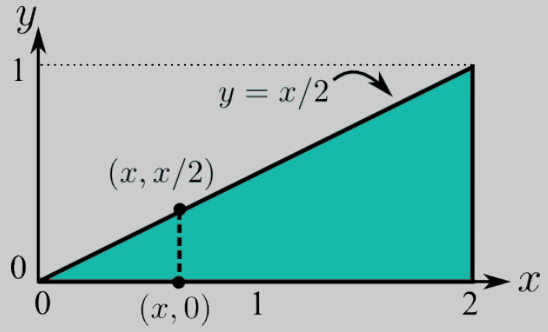
Solution: We see the problem (specifically, the inside integral) specifies that x ranges from $2y$ (a slanted line) to $x = 2$. The second variable, y then varies from $y = 0$ to $y = 1$. Putting this together we get the figure below.



A few of you got the top region, which is almost correct, but note in that case, x varies between 0 and $2y$.

3 2. Rewrite this integral as an integral (with appropriate bounds) in the order $\int\int f\,dy\,dx$.

Solution: We just need now first consider what y varies between. Drawing an arbitrary vertical line, we see it starts at $y = 0$ and ends at $x = 2y$ or $y = x/2$. From there, these slices start at $x = 0$ to $x = 1$, as seen below.



Thus, the resulting integral is

$$\iint f(x,y)\,\mathrm{d}A = \int_0^1 \int_0^{x/2} xy^2\,\mathrm{d}y\,\mathrm{d}x.$$

4 3. Using either the original formulation or the one you found in question 2, evaluate this integral.

Solution: Using the original approach:

$$\begin{aligned}
\iint_D xy^2 dA &= \int_0^1 \left(\int_{2y}^2 xy^2 dx \right) dy \\
&= \int_0^1 \left(\frac{x^2y^2}{2}\Big|_{x=2y}^{x=2} \right) dy \\
&= \int_0^1 \left(2y^2 - \frac{(2y)^2y^2}{2} \right) dy \\
&= \int_0^1 \left(2y^2 - 2y^4 \right) dy \\
&= 2\left[\frac{y^3}{3} - \frac{y^5}{5} \right]_0^1 \\
&= 2\left(\frac{1}{3} - \frac{1}{5} - (0-0) \right) \\
&= 2 \cdot \frac{2}{15} = \frac{4}{15}.
\end{aligned}$$

or, the formulation in question 2:

$$\begin{aligned}
\iint_D xy^2 dA &= \int_0^2 \left(\int_0^{x/2} xy^2 dy \right) dx \\
&= \int_0^2 \left(\frac{x}{3}y^3\Big|_{y=0}^{y=x/2} \right) dx \\
&= \int_0^2 \left(\frac{x}{3}\left(\frac{x}{2}\right)^3 - \frac{x}{3}0^3 \right) dx \\
&= \int_0^2 \frac{x^4}{24} dx \\
&= \frac{x^5}{5 \cdot 24}\Big|_0^2 = \frac{32}{5 \cdot 24} = \frac{4}{15}.
\end{aligned}$$