Name:____ Quiz Score:_____/10

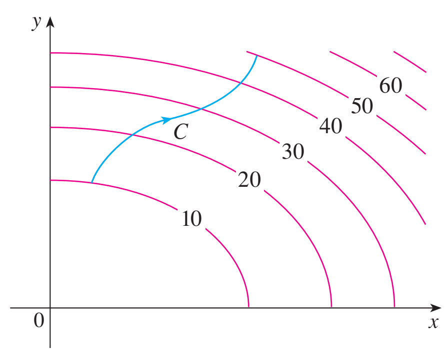
Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

[2] 1. Suppose you're asked to determine the trajectory that requires the least work for a force field **F** to move a particle from one point to another. If you were provided with the additional information that **F** is conservative, how would you respond?

Solution: Although many of you answered something along the lines of Green's Theorem or a statement about a closed loop, this information was *not* provided in the problem.

The answer I was hoping for: given that the Fundamental Theorem of Line Integrals says that the line integral of a conservative vector field *only* depends on the endpoints, the actual trajectory you take does not matter.

[2] 2. Suppose $\mathbf{F} = \nabla f$, a conservative vector field, and C is the curve shown below along with the **contours** of f.



Compute the following and justify your answer:

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

Solution: This is an immediate consequence of the Fundamental Theorem of Line Integrals, which says

$$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a)),$$

where a, b are the starting and ending points of the curve. In this particular case, this is equal to $50 - 10 = 40$.

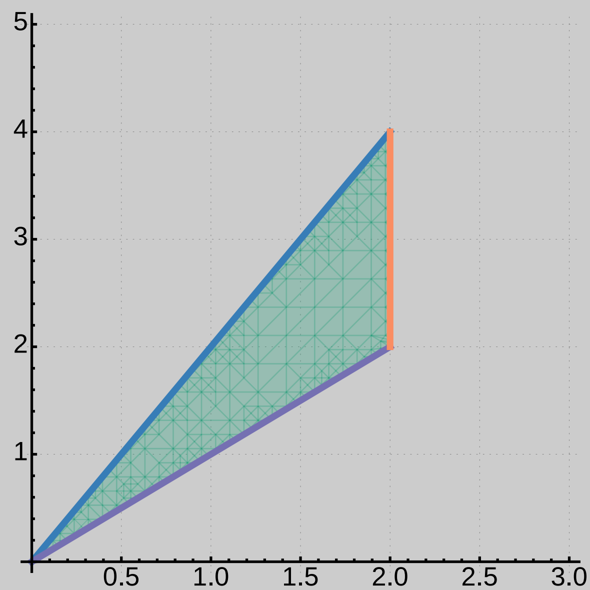
[6] 3. Use Green's Theorem to evaluate the line integral:

$$\oint_C xy^2\,\mathrm{d}x + 2x^2y\,\mathrm{d}y \qquad \text{where } C \text{ is the triangle with vertices } (0,0),\ (2,2),\ (2,4).$$

Solution: Green's Theorem (ignoring all the technical conditions), says that if $\mathbf{F} = \langle P, Q\rangle$, then

$$\oint_C \mathbf{F}\cdot \mathrm{d}\mathbf{r} = \oint_C P\,\mathrm{d}x + Q\,\mathrm{d}y = \iint_D \left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right]\mathrm{d}A.$$

Here, we see that $P = xy^2$ and $Q = 2x^2y$ meaning that $Q_x = 4xy$ and $P_y = 2xy$ and our integrand is $Q_x - P_y = 4xy - 2xy = 2xy$.



To know the bounds of integration, we must draw the region. We could set this up as either type 1 or type 2, but I think type 1 is easier here. Note that x varies between 0 and 2 and then y is bounded by the two sides of the triangle described by $y = 2x$ and $y = x$, thus:

$$\begin{aligned}
\iint_D \left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right]\mathrm{d}A &= \int_0^2 \int_x^{2x} 2xy\,\mathrm{d}y\,\mathrm{d}x \\
&= \int_0^2 \left[xy^2\right]_{y=x}^{y=2x}\mathrm{d}x \\
&= \int_0^2 3x^3\,\mathrm{d}x \\
&= \left[\frac{3}{4}x^4\right]_{x=0}^{x=2} = 12.
\end{aligned}$$

Something worth noting: only one person asked about the orientation of the triangle, which everyone else assumed to be positive. In general, this DOES change the answer!