Name:________________________ Quiz Score:____/10

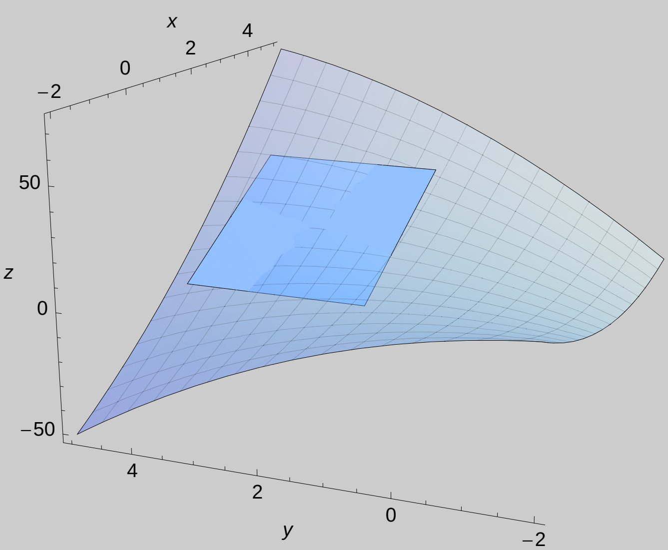
Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

[7] 1. Consider the surface

$$z = f(x, y) = x^2 + 3xy - y^2.$$

Find the equation for the tangent plane at the point $(x, y) = (2, 3)$.

Solution:



We know that the equation of the tangent plane at a point (x_0, y_0, z_0) on $z = f(x, y)$ satisfies

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

Thus, we need to compute each partial derivative, f_x and f_y, where we think of the other variable remaining constant, which leads to:

$$f_x(x, y) = 2x + 3y, \qquad f_y(x, y) = 3x - 2y.$$

We must evaluate these, and the function $f(x, y)$ itself at the point of interest. First note that $f(2, 3) = 13 = z_0$and also

$$f_x(2, 3) = 2(2) + 3(3) = 13, \qquad f_y(2, 3) = 3(2) - 2(3) = 0.$$

Our tangent plane equation is then is simply (note the y terms fall out)

$$z - 13 = 13(x - 2).$$

3 2. Using your answer from question 1, *approximate* the value of z when $(x, y) = (2.05, 2.96)$.

Solution: Recall that the tangent plane is described by

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

We can interpret this as: near the point (x_0, y_0, z_0), the value of $z = f(x, y)$ can be approximated by the value of z on the tangent plane. We can see this by looking at the plot of the tangent plane and the true surface above.

Thus, if we think of moving away slightly from the point (x_0, y_0, z_0) and calling $\delta x = x - x_0$, $\delta y = y - y_0$ and $\delta z = z - z_0$, we get the form of the differential:

$$\delta z = f_x(x_0, y_0)\delta x + f_y(x_0, y_0)\delta y.$$

In our particular case, we have that $\delta x = 2.05 - 2 = 0.05$ and $\delta y = 2.96 - 3 = -0.04$, meaning we can now approximate the true value of z by:

$$z - 13 \approx 13(0.05) + 0(-0.04) \qquad \Longrightarrow \qquad z \approx 13 + 13(.05) = 13.65.$$