Name:________________________________________ Quiz Score:_____/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

[2] 1. Consider vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$. Which of the following operations **do not** make sense? There may be more than one answer.

1. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$.
2. $\mathbf{a} \times (\mathbf{b} \cdot \mathbf{c})$.
3. $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$.
4. $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$.

**Solution:** Ultimately this question boils down to: what does it makes sense to take the dot or cross product of? Both of these require two vectors input, but

$$\mathbf{a} \cdot \mathbf{b} = \text{scalar}, \qquad \mathbf{a} \times \mathbf{b} = \text{vector}.$$

Thus, we see that in 2. $\mathbf{b} \cdot \mathbf{c}$ is a scalar, meaning we cannot take the cross product of it. A similar situation occurs in 4.

[4] 2. Compute the angle between the two vectors

$$\mathbf{v}_1 = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}, \qquad \mathbf{v}_2 = \langle 4, 0, -3 \rangle.$$

**Solution:** The definition of the dot product says

$$\mathbf{v}_1 \cdot \mathbf{v}_2 = \|\mathbf{v}_1\| \, \|\mathbf{v}_2\| \cos\theta,$$

where $\theta$ is the angle between the two vectors, meaning that we can recover the angle by rearranging to yield

$$\theta = \cos^{-1}\left(\frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{\|\mathbf{v}_1\| \, \|\mathbf{v}_2\|}\right)$$

Computing the magnitudes quantities, we find

$$\|\mathbf{v}_1\| = \sqrt{1^2 + 2^2 + (-2)^2} = 3, \qquad \|\mathbf{v}_2\| = \sqrt{4^2 + 0^2 + (-3)^2} = 5,$$

and lastly, the dot product itself is

$$\mathbf{v}_1 \cdot \mathbf{v}_2 = 1(4) + 1(0) + (-2)(-3) = 10.$$

Thus, combining all of these parts, we're left with

$$\theta = \cos^{-1}\left(\frac{10}{3 \cdot 5}\right) = \cos^{-1}\left(\frac{2}{3}\right).$$

4 3. Compute $\mathbf{u}_1 \times \mathbf{u}_2$, where

$$\mathbf{u}_1 = \langle 1, 1, -1 \rangle, \qquad \mathbf{u}_2 = \langle 2, 4, 6 \rangle.$$

**Solution:** We'll use the determinant method for computing the cross product, which states, we must compute

$$\mathbf{u}_1 \times \mathbf{u}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & -1 \\ 2 & 4 & 6 \end{vmatrix} = \mathbf{i} \begin{vmatrix} 1 & -1 \\ 4 & 6 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 1 & -1 \\ 2 & 6 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} = (16 + 1 \cdot 4)\mathbf{i} - (1 \cdot 6 + 1 \cdot 2)\mathbf{j} + (1 \cdot 4 - 1 \cdot 2)\mathbf{k}$$

which, after doing a little bit of algebra, yields the final answer

$$\mathbf{u}_1 \times \mathbf{u}_2 = 10\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}.$$