

Name: _____

Quiz Score: ____/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

- 5 1. Consider the following function:

$$f(x, y) = x^2 + 3xy + y^2 + \cos(xy).$$

Compute the first partial derivatives in each variable. That is, compute $\partial f/\partial x$ and $\partial f/\partial y$.

Solution:

As we've discussed many times now, to take $\partial f/\partial x$, we simply treat y as a constant and take the standard x derivative, resulting in:

$$\frac{\partial f}{\partial x} = 2x + 3y - y \sin xy.$$

Similarly, for y , we treat x as a constant:

$$\frac{\partial f}{\partial y} = 3y + 2y - x \sin xy.$$

- 5 2. Determine the value of the following limit (if it exists):

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}.$$

If the limit exists, prove it. If the limit does not exist, prove this instead. *Hint:* which is easier?

Solution: As the problem hints at, it's much easier to show a limit does not exist than showing that it does exist. To do so, we simply need to find 2 paths approaching our point $(0,0)$ that result in different limits. Therefore, if we consider either the horizontal path ($y = 0$), we get:

$$\lim_{x \rightarrow 0} \frac{0 \cdot x}{x^2 + 0^2} = \lim_{x \rightarrow 0} 0 = 0,$$

or the vertical path ($x = 0$), we get:

$$\lim_{y \rightarrow 0} \frac{0 \cdot y}{0^2 + y^2} = \lim_{y \rightarrow 0} 0 = 0.$$

Therefore, we need to consider some other path that might result in a different limit. From class, consider the angled approach along $y = x$. We now replace y in the limit with x and see:

$$\lim_{x \rightarrow 0} \frac{x \cdot x}{x^2 + x^2} = \lim_{x \rightarrow 0} \frac{x^2}{2x^2} = \lim_{x \rightarrow 0} \frac{1}{2} = \frac{1}{2}.$$

Thus, since we've found two paths where the limits differ, the limit does not exist. If you're curious, this is the figure of the plot from the book:

