

Name: \_\_\_\_\_

Quiz Score: \_\_\_\_\_/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

1. Consider the two following vectors:

$$\mathbf{a} = \langle 1, 2, 3 \rangle, \quad \mathbf{b} = \langle -1, 4, 2 \rangle.$$

- 3 (a) Compute  $\mathbf{a} \cdot \mathbf{b}$ .

**Solution:** Our method for the dot product is to take the product of the first element of each and add it to the product of the second element of each, and then add that to the product of the third element of each. Thus

$$\mathbf{a} \cdot \mathbf{b} = 1(-1) + 2(4) + 3(2) = 13.$$

- 3 (b) What is the angle between  $\mathbf{a}$  and  $\mathbf{b}$ ? *Note:* you don't have to simplify.

**Solution:** Here, we use the other dot product formula

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos \theta.$$

We know the left hand side from part (a), thus we just need to compute the magnitudes of each of the vectors

$$\|\mathbf{a}\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}, \quad \|\mathbf{b}\| = \sqrt{(-1)^2 + 4^2 + 2^2} = \sqrt{21}.$$

We can now use this to obtain  $\theta$

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} = \frac{13}{\sqrt{14}\sqrt{21}},$$

meaning that the angle itself is

$$\theta = \cos^{-1} \frac{13}{\sqrt{14}\sqrt{21}}.$$

- 4 2. Consider the same two vectors:

$$\mathbf{a} = \langle 1, 2, 3 \rangle, \quad \mathbf{b} = \langle -1, 4, 2 \rangle.$$

Compute  $\mathbf{a} \times \mathbf{b}$ .

**Solution:** Here, we utilize our  $3 \times 3$  determinant to calculate the cross product

$$\begin{aligned} \mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ -1 & 4 & 2 \end{vmatrix} \\ &= \mathbf{i} \begin{vmatrix} 2 & 3 \\ 4 & 2 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 1 & 2 \\ -1 & 4 \end{vmatrix} \\ &= \mathbf{i}[2(2) - (4)3] - \mathbf{j}[1(2) - (-1)3] + \mathbf{k}[1(4) - (-1)2] \\ &= -8\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}. \end{aligned}$$