Name:____________________________________ Quiz Score:_____/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

1. Consider the following matrix:

$$\mathbf{A} = \begin{bmatrix} 7 & 9 \\ 5 & 7 \end{bmatrix}.$$

[4] (a) Find $\mathbf{A}^{-1}$.

Solution: We actually know a number of ways to invert 2×2 matrices. The easiest says that if $\mathbf{A}$ is invertible, then the following relationship is true:

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \Longrightarrow \qquad \mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

In our particular case, $ad - bc = 49 - 45 = 4$, and therefore

$$\mathbf{A}^{-1} = \frac{1}{4} \begin{bmatrix} 7 & -9 \\ -5 & 7 \end{bmatrix}.$$

Note that if this were a 3×3 matrix, we don't have a nice form like this, so we would need to do the augmented row-reduction by starting with augmenting the identity matrix and row reducing until we get the identity matrix on the left:

$$\left[\begin{array}{cc|cc} 7 & 9 & 1 & 0 \\ 5 & 7 & 0 & 1 \end{array}\right] \xrightarrow{R_1/7} \left[\begin{array}{cc|cc} 1 & 9/7 & 1/7 & 0 \\ 5 & 7 & 0 & 1 \end{array}\right] \xrightarrow{-5R_1+R_2} \left[\begin{array}{cc|cc} 1 & 9/7 & 1/7 & 0 \\ 0 & 7 - \frac{45}{7} & -5/7 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 9/7 & 1/7 & 0 \\ 0 & 4/7 & -5/7 & 1 \end{array}\right] \xrightarrow{(-9/4)R_2+R_1} \left[\begin{array}{cc|cc} 1 & 0 & 7/4 & -9/4 \\ 0 & 4/7 & -5/7 & 1 \end{array}\right] \xrightarrow{(7/4)R_2} \left[\begin{array}{cc|cc} 1 & 0 & 7/4 & -9/4 \\ 0 & 1 & -5/4 & 7/4 \end{array}\right].$$

[4] (b) Use $\mathbf{A}^{-1}$ to solve $\mathbf{Ax} = \mathbf{b}$ for the following $\mathbf{b}$:

$$\mathbf{b} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

Solution: Once we have $\mathbf{A}^{-1}$, we know that it is exists and therefore our solution is just

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b} = \frac{1}{4} \begin{bmatrix} 7 & -9 \\ -5 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 7 \cdot 3 + (-9) \cdot 2 = 3 \\ (-5) \cdot 3 + 7 \cdot 2 = -1 \end{bmatrix}.$$

2 2. Which of the following statements is **not** equivalent to the others? Here, $\mathbf{A}$ is an $n \times n$ matrix.

1. $\mathbf{A}$ is invertible.
2. $\mathbf{A}$ is row equivalent to $\mathbf{I}$.
3. $\mathbf{Ax} = 0$ has infinitely many solutions.
4. $\mathbf{Ax} = \mathbf{b}$ has a unique solution.

Solution: $\mathbf{Ax} = 0$ has infinitely many solutions is the **incompatible** statement. We know the statement that is equivalent to the others is actually:

$\mathbf{Ax} = 0$ only has the trivial solution