Name:____________________________________ Quiz Score:______/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

8 1. Solve (by finding a homogeneous and particular solution) the following differential equation:

$$y'' - 2y' - 3y = 36e^{5x}, \qquad y(0) = 9, \qquad y'(0) = 25.$$

Solution: We know that in general, the solution to this problem will be of the form

$$y(x) = y_c(x) + y_p(x),$$

where $y_c(x)$ is the complementary solution (that solves the corresponding homogeneous equation) and $y_p(x)$ is some particular solution to the non-homogeneous equation. To get the correct y_p, we've discussed that we should first find y_c. Thus, we consider the homogeneous equation:

$$y_c'' - 2y_c' - 3y_c = 0.$$

We know we can solve this constant coefficient equation by solving the characteristic equation for the roots:

$$r^2 - 2r - 3 = (r-3)(r+1) = 0 \qquad \Longrightarrow r_1 = 3, \quad r_2 = -1.$$

Thus, our complementary solution is of the form

$$y_c(x) = c_1 e^{3x} + c_2 e^{-x}.$$

Next, we turn to the particular solution. We want to guess $y_p(x) = Ae^{5x}$ and we see that this is *not* a duplication from the homogeneous part, so we can indeed guess and plug it in:

$$y_p'' - 2y_p' - 3y_p = (25A - 10A - 3A)\, e^{5x} = 36e^{5x}$$

thus, we see that we have the same exponential on the left and right if $25A - 10A - 3A = 36$ or $A = 3$. Thus we know $y_p(x) = 3e^{5x}$. Our solution, in total is then

$$y(x) = y_c(x) + y_p(x) = c_1 e^{3x} + c_2 e^{-x} + 3e^{5x}.$$

How do we determine c_1, c_2? We need to enforce the two initial conditions, $y(0) = 9$ and $y'(0) = 25$. First, we have

$$y(0) = 9 = c_1 + c_2 + 3, \qquad y'(0) = 25 = 3c_1 - c_2 + 15.$$

We can add these together to see that $4c_1 + 18 = 34$, so $c_1 = 4$ and therefore, plugging this back into either yields $c_2 = 2$. From this, all coefficients are determined and in total, our solution is

$$y(x) = 4e^{3x} + 2e^{-x} + 3e^{5x}.$$

2 2. Assuming there is no duplication from the homogeneous component, write down an appropriate form for a particular solution for the differential equation:

$$y'' + p(x)y' + q(x)y = -e^{-2x}(3 - 5x + 7x^2)\cos(9x).$$

Note that you do **not** need to solve for the coefficients.

Solution: To determine the guess here, we see that we effectively have three parts: an exponential multiplied by a second degree polynomial, multiplied by a cosine. Our guess should be of the exact same form, with the caveat that if we ever have a cosine (or sine) we need to include both sines and cosines of the same type.

Thus, the guess is of the form:

$$y_p(x) = e^{-2x}\left(A_0 + A_1x + A_2x^2\right)\cos 9x + e^{-2x}\left(B_0 + B_1x + B_2x^2\right)\sin 9x.$$

Us needing two separate polynomials for each trig term is an important thing to note.