

Name: \_\_\_\_\_

Quiz Score: \_\_\_\_/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

1. For the following problems, state whether the statement is **true** or **false**. If it is **false**, explain why.

- 2 (a) Any three vectors in  $\mathbb{R}^3$  form a basis. That is, any vector  $\mathbf{w} \in \mathbb{R}^3$  can be written as a linear combination of any other three vectors.

**Solution:** This is **false**. The correct statement is actually:

Any three **linearly independent** vectors in  $\mathbb{R}^3$  form a basis. Think about if all three of your vectors were scalar multiples of each other or something similarly problematic. Could you get every other vector in  $\mathbb{R}^3$  from this? No.

- 2 (b) If a set of three vectors are linearly dependent in  $\mathbb{R}^3$ , they are all scalar multiples of each other.

**Solution:** This is also **false**. Although we say a *pair* of vectors is linearly dependent if they are scalar multiples of each other, we say that a set (three or more) vectors is linearly dependent if one can be written as a linear combination of the others.

Note that this also follows from our algebraic definition that says, there exists  $c_1, c_2, c_3 \neq 0$  such that

$$0 = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3,$$

from which we can, for example, rearrange to find

$$\mathbf{v}_1 = -\frac{c_2}{c_1}\mathbf{v}_2 - \frac{c_3}{c_1}\mathbf{v}_3,$$

which we can do since  $c_1 \neq 0$ .

- 2 (c) The only subspaces of  $\mathbb{R}^3$  are lines and planes through the origin.

**Solution:** This is **false**. As people pointed out in class, the subsets  $\{\mathbf{0}\}$  and all of  $\mathbb{R}^3$  are technically subspaces but aren't lines or planes. This is a technicality I forgot when writing the problem, so aside from those two, this is **true**.

See the argument from class where every subspace either has a pair of linearly independent vectors or it does not, resulting in only planes and lines.

- 4 2. Are the following three vectors in  $\mathbb{R}^3$  linearly independent or dependent?

$$\mathbf{v}_1 = (1, 0, 0), \quad \mathbf{v}_2 = (6, 5, 0), \quad \mathbf{v}_3 = (10, 8, 7).$$

*Hint:* we know a shortcut based on this special type of matrix.

**Solution:** The method for testing linear independence of three vectors we've established is to put them in a matrix (as columns, say) and take the determinant:

$$D = \begin{vmatrix} \vdots & \vdots & \vdots \\ \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \\ \vdots & \vdots & \vdots \end{vmatrix} = \begin{vmatrix} 1 & 6 & 10 \\ 0 & 5 & 8 \\ 0 & 0 & 7 \end{vmatrix} = 35.$$

How did I know the determinant was 35? Notice this is a **upper triangular matrix**, thus the determinant is just the product of the diagonal entries, so  $1 \times 5 \times 7 = 35$ !

Our theorem says that the three vectors are linearly independent if and only if the determinant  $\neq 0$ , thus these three vectors are **linearly independent**.