Name:_____   Quiz Score:_____/10

> Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

1. Consider the following differential equation and initial condition

$$\frac{dy}{dx} = (x-2)^2, \qquad y(2) = 1.$$

$\boxed{6}$ (a) Find a function $y(x)$ that satisfies the above differential equation and initial condition. That is, solve the initial value problem.

> **Solution:** We see that this differential equation is of the form
>
> $$\frac{dy}{dx} = f(x),$$
>
> which means that we can simply integrate both sides with respect to x. Doing so yields
>
> $$\int \frac{dy}{dx}\, dx = \int (x-2)^2 \, dx$$
>
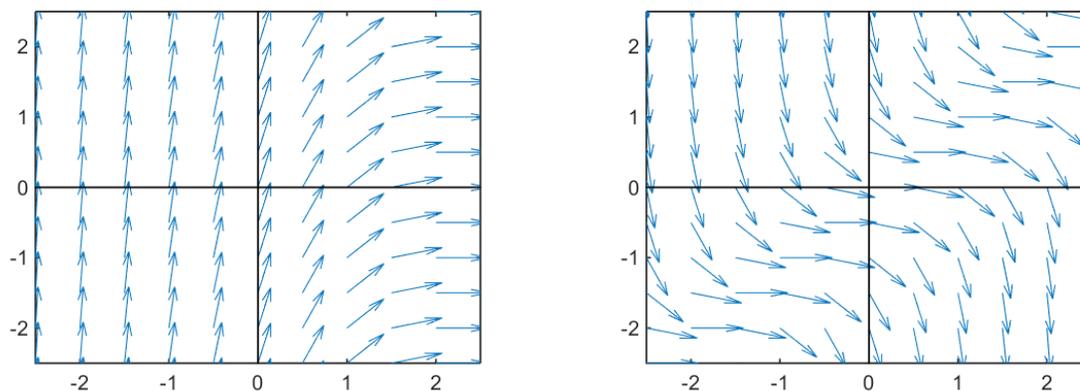> $$y(x) = \frac{1}{3}(x-2)^3 + C.$$
>
> Finally, we can solve for C by plugging in $x = 2$, $y = 1$, to yield
>
> $$1 = \frac{1}{3}(2-2)^3 + C \implies C = 1.$$
>
> In summary, solution is then
>
> $$y(x) = \frac{1}{3}(x-2)^3 + 1.$$

2   (b) Which of the following is the appropriate slope field for the differential equation? **Why?**

> **Solution:** The correct answer is the **left**. There are a variety of ways you could argue for this. For one, $(x-2)^2$ is always a positive number, meaning the slope must always be positive. Clearly the right has negative slopes, meaning we can eliminate it. Also, we see that the derivative is 0 when $x = 2$, meaning we have a horizontal line, which the left also has but the right does not.

2   (c) We can can be sure that the the the solution you found in part (a) is unique. **Why?**
         Hint: just write the conditions of our existence and uniqueness theorem and show why they are satisfied here.

> **Solution:** The existence and uniqueness theorem requires that, on some rectangle R in the $x - y$ plane, that $f(x, y)$ and $\partial f / \partial y$ be continuous. Here, we see that
>
> $$f(x, y) = (x - 2)^2.$$
>
> Clearly this is continuous, but how do we compute $\partial f / \partial y$? There are no y's in the equation, thus $\partial f / \partial y \equiv 0$, which is also continuous. Thus, our rectangle R can be as large as we want, which definitely includes our initial point $(2, 1)$, and therefore we are guaranteed uniqueness for some interval around our initial point.
>
> As we've emphasized many times: despite the fact that we can make R as large as we'd like, we don't know how large the interval is that we have uniqueness. For this problem, it turns out the solution is unique everywhere, but that isn't a result of our theorem. We only know that we have *some* interval around our initial condition that uniqueness holds.