5. $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} -9 & 1 \\ -10 & 12 \end{bmatrix}$; $\quad \begin{bmatrix} -4 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 8 \\ 11 & 5 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 0 & -3 \\ 3 & 2 & 4 \\ 2 & -3 & 5 \end{bmatrix}\begin{bmatrix} 7 & -4 & 3 \\ 1 & 5 & -2 \\ 0 & 3 & 9 \end{bmatrix} = \begin{bmatrix} 7 & -13 & -24 \\ 23 & 10 & 41 \\ 11 & -8 & 57 \end{bmatrix}$

$\begin{bmatrix} 7 & -4 & 3 \\ 1 & 5 & -2 \\ 0 & 3 & 9 \end{bmatrix}\begin{bmatrix} 1 & 0 & -3 \\ 3 & 2 & 4 \\ 2 & -3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -17 & -22 \\ 12 & 16 & 7 \\ 27 & -21 & 57 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 26 \end{bmatrix}$; $\quad \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 4 & 8 & 12 \\ 5 & 10 & 15 \end{bmatrix}$

8. $\begin{bmatrix} 1 & 0 & 3 \\ 2 & -5 & 4 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ -1 & 4 \\ 6 & 5 \end{bmatrix} = \begin{bmatrix} 21 & 15 \\ 35 & 0 \end{bmatrix}$; $\quad \begin{bmatrix} 3 & 0 \\ -1 & 4 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 & 3 \\ 2 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 9 \\ 7 & -20 & 13 \\ 16 & -25 & 38 \end{bmatrix}$

9. $\begin{bmatrix} 0 & -2 \\ 3 & 1 \\ -4 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ -22 \end{bmatrix}$ but the product $\begin{bmatrix} 3 \\ -2 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 3 & 1 \\ -4 & 5 \end{bmatrix}$ is not defined.

10. $\mathbf{AB} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 & 4 \\ 3 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 13 \\ 5 & -6 & 31 \end{bmatrix}$ but the product $\mathbf{BA}$ is not defined.

11. $\mathbf{AB} = \begin{bmatrix} 3 & -5 \end{bmatrix}\begin{bmatrix} 2 & 7 & 5 & 6 \\ -1 & 4 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 1 & 5 & 3 \end{bmatrix}$ but the product $\mathbf{BA}$ is not defined.

12. Neither product matrix $\mathbf{AB}$ or $\mathbf{BA}$ is defined.

13. $\mathbf{A(BC)} = \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix}\left(\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}\right) = \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 10 & 17 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 32 & 51 \\ -2 & -17 \end{bmatrix}$

$\mathbf{(AB)C} = \left(\begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}\right)\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 16 \\ -14 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 32 & 51 \\ -2 & -17 \end{bmatrix}$

In Problems 1–8 we first give the inverse matrix $\mathbf{A}^{-1}$ and then calculate the solution vector $\mathbf{x}$.

1. $\mathbf{A}^{-1} = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$; $\mathbf{x} = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

2. $\mathbf{A}^{-1} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$; $\mathbf{x} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -26 \\ 11 \end{bmatrix}$

3. $\mathbf{A}^{-1} = \begin{bmatrix} 6 & -7 \\ -5 & 6 \end{bmatrix}$; $\mathbf{x} = \begin{bmatrix} 6 & -7 \\ -5 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 33 \\ -28 \end{bmatrix}$

4. $\mathbf{A}^{-1} = \begin{bmatrix} 17 & -12 \\ -7 & 5 \end{bmatrix}$; $\mathbf{x} = \begin{bmatrix} 17 & -12 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 25 \\ -10 \end{bmatrix}$

5. $\mathbf{A}^{-1} = \frac{1}{2}\begin{bmatrix} 4 & -2 \\ -5 & 3 \end{bmatrix}$; $\mathbf{x} = \frac{1}{2}\begin{bmatrix} 4 & -2 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 5 \\ 6 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 8 \\ -7 \end{bmatrix}$

6. $\mathbf{A}^{-1} = \frac{1}{3}\begin{bmatrix} 6 & -7 \\ -3 & 4 \end{bmatrix}$; $\mathbf{x} = \frac{1}{3}\begin{bmatrix} 6 & -7 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 10 \\ 5 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 25 \\ -10 \end{bmatrix}$

7. $\mathbf{A}^{-1} = \frac{1}{4}\begin{bmatrix} 7 & -9 \\ -5 & 7 \end{bmatrix}$; $\mathbf{x} = \frac{1}{4}\begin{bmatrix} 7 & -9 \\ -5 & 7 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 3 \\ -1 \end{bmatrix}$

8. $\mathbf{A}^{-1} = \frac{1}{5}\begin{bmatrix} 10 & -15 \\ -5 & 8 \end{bmatrix}$; $\mathbf{x} = \frac{1}{5}\begin{bmatrix} 10 & -15 \\ -5 & 8 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 25 \\ -11 \end{bmatrix}$

In Problems 9–22 we give at least the first few steps in the reduction of the augmented matrix whose right half is the identity matrix of appropriate size. We wind up with its echelon form, whose left half is an identity matrix and whose right half is the desired inverse matrix.

9. $\begin{bmatrix} 5 & 6 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{bmatrix} \xrightarrow{R1-R2} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 4 & 5 & 0 & 1 \end{bmatrix} \xrightarrow{R2-4R1} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & -4 & 5 \end{bmatrix}$

$\xrightarrow{R1-R2} \begin{bmatrix} 1 & 0 & 5 & -6 \\ 0 & 1 & -4 & 5 \end{bmatrix}$; thus $\mathbf{A}^{-1} = \begin{bmatrix} 5 & -6 \\ -4 & 5 \end{bmatrix}$

10. $\begin{bmatrix} 5 & 7 & 1 & 0 \\ 4 & 6 & 0 & 1 \end{bmatrix} \xrightarrow{R1-R2} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 4 & 6 & 0 & 1 \end{bmatrix} \xrightarrow{R2-4R1} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 2 & -4 & 5 \end{bmatrix}$

$$\xrightarrow{R2-2R1} \begin{bmatrix} 1 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & -2 & 0 \\ 3 & 7 & 9 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R3-3R1} \begin{bmatrix} 1 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & -2 & 0 \\ 0 & -2 & 3 & 0 & -3 & 1 \end{bmatrix}$$

$$\xrightarrow{R3+2R2} \cdots \rightarrow \begin{bmatrix} 1 & 0 & 0 & -13 & 42 & -5 \\ 0 & 1 & 0 & 3 & -9 & 1 \\ 0 & 0 & 1 & 2 & -7 & 1 \end{bmatrix}; \quad \text{thus } \mathbf{A}^{-1} = \begin{bmatrix} -13 & 42 & -5 \\ 3 & -9 & 1 \\ 2 & -7 & 1 \end{bmatrix}$$

14. $$\begin{bmatrix} 3 & 5 & 6 & 1 & 0 & 0 \\ 2 & 4 & 3 & 0 & 1 & 0 \\ 2 & 3 & 5 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R1-R2} \begin{bmatrix} 1 & 1 & 3 & 1 & -1 & 0 \\ 2 & 4 & 3 & 0 & 1 & 0 \\ 2 & 3 & 5 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R2-R3} \begin{bmatrix} 1 & 1 & 3 & 1 & -1 & 0 \\ 0 & 1 & -2 & 0 & 1 & -1 \\ 2 & 3 & 5 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R3-2R1} \begin{bmatrix} 1 & 1 & 3 & 1 & -1 & 0 \\ 0 & 1 & -2 & 0 & 1 & -1 \\ 0 & 1 & -1 & -2 & 2 & 1 \end{bmatrix}$$

$$\xrightarrow{R3-R2} \cdots \rightarrow \begin{bmatrix} 1 & 0 & 0 & 11 & -7 & -9 \\ 0 & 1 & 0 & -4 & 3 & 3 \\ 0 & 0 & 1 & -2 & 1 & 2 \end{bmatrix}; \quad \text{thus } \mathbf{A}^{-1} = \begin{bmatrix} 11 & -7 & -9 \\ -4 & 3 & 3 \\ -2 & 1 & 2 \end{bmatrix}$$

15. $$\begin{bmatrix} 1 & 1 & 5 & 1 & 0 & 0 \\ 1 & 4 & 13 & 0 & 1 & 0 \\ 3 & 2 & 12 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R2-R1} \begin{bmatrix} 1 & 1 & 5 & 1 & 0 & 0 \\ 0 & 3 & 8 & -1 & 1 & 0 \\ 3 & 2 & 12 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R3-3R1} \begin{bmatrix} 1 & 1 & 5 & 1 & 0 & 0 \\ 0 & 3 & 8 & -1 & 1 & 0 \\ 0 & -1 & -3 & -3 & 0 & 1 \end{bmatrix} \xrightarrow{R2+2R3} \begin{bmatrix} 1 & 1 & 5 & 1 & 0 & 0 \\ 0 & 1 & 2 & -7 & 1 & 2 \\ 0 & -1 & -3 & -3 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R3+R2} \cdots \rightarrow \begin{bmatrix} 1 & 0 & 0 & -22 & 2 & 7 \\ 0 & 1 & 0 & -27 & 3 & 8 \\ 0 & 0 & 1 & 10 & -1 & -3 \end{bmatrix}; \quad \text{thus } \mathbf{A}^{-1} = \begin{bmatrix} -22 & 2 & 7 \\ -27 & 3 & 8 \\ 10 & -1 & -3 \end{bmatrix}$$

16. $$\begin{bmatrix} 1 & -3 & -3 & 1 & 0 & 0 \\ -1 & 1 & 2 & 0 & 1 & 0 \\ 2 & -3 & -3 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R2+R1} \begin{bmatrix} 1 & -3 & -3 & 1 & 0 & 0 \\ 0 & -2 & -1 & 1 & 1 & 0 \\ 2 & -3 & -3 & 0 & 0 & 1 \end{bmatrix}$$