Math 2250 HW2

[1.4] 8, 26
[1.5] 14, 37.

[1.4.8] $\frac{dy}{dx} = 2x\sec y$

$$dy \cdot \frac{1}{\sec y} = 2x\,dx$$

$$\cos y\,dy = 2x\,dx$$

$$\int \cos y\,dy = \int 2x\,dx$$

$$\sin(y) = x^2 + C.$$

$$\boxed{y = \sin^{-1}(x^2 + C)}$$

[1.4.26] $\frac{dy}{dx} = 2xy^2 + 3x^2y^2,\ y(1) = -1$

$$= y^2(2x + 3x^2).$$

$$y^{-2}\,dy = (2x + 3x^2)\,dx$$

$$\int y^{-2}\,dy = \int (2x+3x^2)\,dx$$

$$-y^{-1} = x^2 + x^3 + C$$

$$-\frac{1}{y} = x^2 + x^3 + C$$

$$y = \frac{-1}{x^2+x^3+C}$$

[illegible]

but $y(1) = -1$

$$-1 = \frac{-1}{1+1+C}$$

$$\Rightarrow C = -1$$

$$y = \frac{-1}{x^2+x^3-1}$$

[1.5.14] $xy' - 3y = x^3$, $y(1) = 10$.

$$y' - \frac{3}{x}y = x^2, \text{ so } P(x) = -\frac{3}{x}, \quad Q(x) = x^2$$

$$y' + P(x)y = Q(x)$$

$$u = e^{\int P(x)\,dx} = e^{\int \frac{-3}{x}\,dx} = e^{-3\ln x}$$

$$= x^{-3}.$$

$$\frac{d}{dx}\{u(x)y\} = uQ$$

$$\frac{d}{dx}\{x^{-3}y(x)\} = x^{-3}x^2 = \frac{1}{x}$$

Integrate both sides:

$$x^{-3}y(x) = \ln(x) + C.$$

$$y(x) = x^3\ln(x) + Cx^3.$$

$$y(1) = 10 \Rightarrow$$

$$10 = 0 + C1^3$$

$$C \Rightarrow 10.$$

$$y(x) = x^3\ln(x) + 10x^3$$

[1.5.37]

$r_{in} = 5$ gal/s, $c_{in} = 1$ lb/gal.

400 gallon tank.

100 gal initially.

50 lb salt.

$x(0) = 50$ lb

$r_{out} = 3$ gal/s

$\frac{dx}{dt}$ = rate of salt in − rate of salt out

$= r_{in} \cdot c_{in} - r_{out} c_{out}$

What is c_{out}? $\frac{x(t)}{V(t)}$.

$V(t)$? 5 gal in − 3 gal out.

2 gal/s net gain.

$V(t) = 100 + 2t$.

so,

$$\frac{dx}{dt} = 5 \cdot \frac{1}{100+2t} - \frac{x}{100+2t} \cdot 3.$$

$$= 5 - \frac{3x}{100+2t}, \qquad x(0) = 50.$$

Solve:

$$\frac{dx}{dt} + \frac{3}{100+2t} x = 5.$$

$$P = \frac{3}{100+2t}, \quad \mu = e^{\int P(t)\,dt} = e^{\frac{3}{2}\log(t+50)}$$

$$= (t+50)^{3/2}$$

$$\frac{d}{dt}\left\{ (t+50)^{3/2} x(t) \right\} = 5(t+50)^{3/2}.$$

Integrate;

$$x(t)(t+50)^{3/2} = 2(t+50)^{5/2} + C.$$

$$x(t) = 2(t+50) + C(t+50)^{-3/2}$$

$x(0) = 50$.

$$50 = 100 + \frac{C}{50^{3/2}}$$

$$C = -12500\sqrt{2}$$

$$x(t) = (100 + 2t) - \frac{12500\sqrt{2}}{(t+50)^{3/2}}$$

When is tank full?

$V(t) = 400 = 100 + 2t$

$\Rightarrow t = 150$

$x(150)$

$= \frac{1575}{4} = 393.75$ lbs