2250-10 HW1 [1.1] 4, 22 [1.2] 6, 20 [1.3] 6, 12

[1.1.4] $y'' = 9y$, $y_1 = e^{3x}$, $y_2 = e^{-3x}$

$$y_1' = 3e^{3x}$$

$$y_1'' = 3(3)e^{3x} = 9e^{3x} = 9y_1 \checkmark$$

$$y_2' = -3e^{-3x}$$

$$y_2' = (-3)(-3)e^{-3x} = 9e^{-3x} = 9y_2 \checkmark$$

[1.1.22] $e^y y' = 1$, $y(x) = \ln(x+C)$ $y(0) = 0$,

First check IC:

$$y(0) = \ln(c) = 0$$

$$\underline{c = e^0 = 1.}$$

Check DE:

$$y' = \frac{1}{x+C}, \quad e^y y' = e^{\ln(x+c)} \frac{1}{x+c} = \frac{x+c}{x+c} = 1 \checkmark$$

$\boxed{1.2.6}$ $\frac{dy}{dx} = x\sqrt{x^2+9}$, $y(-4)=0$.

$$\int \frac{dy}{dx}\,dx = y(x).$$

$$\int x\sqrt{x^2+9}\,dx,$$

$u = x^2+9$
$du = 2x\,dx$.
$\frac{du}{2} = x\,dx$.

$$\int \frac{u^{1/2}}{2}\,du$$

$$= \frac{u^{3/2}}{2}\cdot\frac{2}{3} + C = \frac{u^{3/2}}{3} + C$$

$$= \frac{(x^2+9)^{3/2}}{3} + C.$$

$y(-4)=0$.

$$0 = \frac{((-4)^2+9)^{3/2}}{3} + C.$$

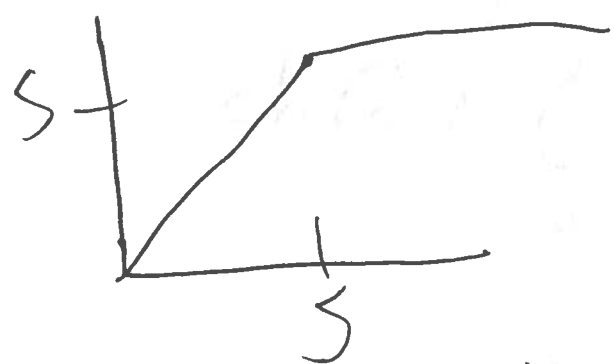
$$= \frac{(25)^{3/2}}{3} + C \Rightarrow C = -\frac{125}{3}.$$

$$\boxed{y(x) = \frac{(x^2+9)^{3/2}}{3} - \frac{125}{3}}$$

[1.2, 20] ③

We can do this either geometrically or algebraically.

Geometrically:



$x(t)$ = area under curve up until t.

$0 < t < 5$:

$x(t)$: t, t $= \frac{t^2}{2}$.

$t > 5$:

$x(t) =$ 5 5 + 5 $t-5$ $= 12.5 + 5(t-5)$

Algebraically we see:

$$v(t) = \begin{cases} t & 0 < t < 5 \\ 5 & t > 5 \end{cases}$$

so:

$$x(t) = x_0 + \int_0^t v(s)\,ds.$$

$0 < t < 5$:

$$= 0 + \int_0^t s\,ds = t^2/2.$$

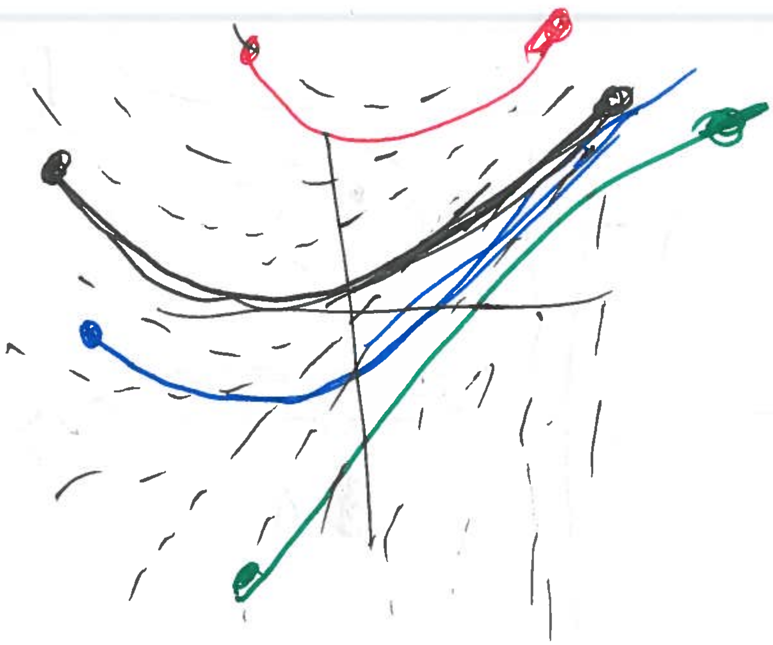
$t > 5$:

$$x(t) = x(5) + \int_5^t v(s)\,ds$$

$$= 12.5 + \int_5^t 5 = 25 + 5(t-5).$$

⑤

[1.3.6]



[1.3.12] $\frac{dy}{dx} = x\ln y, \quad y(1) = 1.$

$f(x,y) = x\ln y$

We must find an R where

$f(x,y), \frac{\partial f}{\partial y}(x,y)$ are continuous.

f is continuous for $y>0$.

$\frac{\partial f}{\partial y} = \frac{x}{y}$, also continuous for $y>0$.

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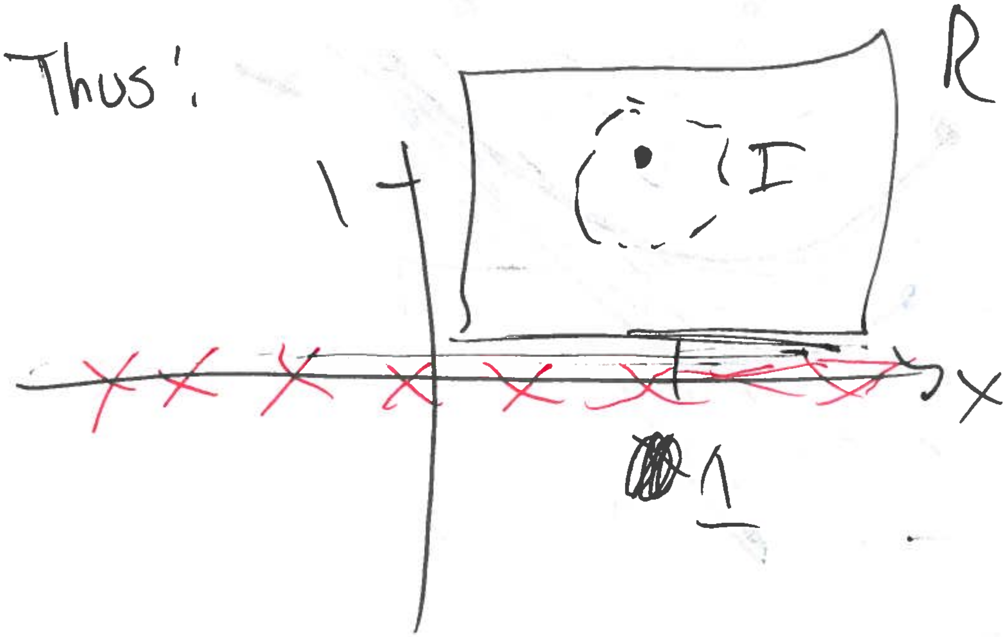
Thus:



We can make our rectangle R as large as we'd like so long as it doesn't include $y=0$.

In R, ~~[illegible]~~ around our initial point $(1,1)$ some solution is guaranteed to exist & be unique in some interval I around $x=1$.

We don't know how large of an interval!