Name:______________________________ Quiz Score:_____/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

1. Consider the following function and corresponding interval:

$$f(x) = 6x^{4/3} - 3x^{1/3}, \qquad x \in [-1, 1]$$

2 (a) Compute the critical numbers of $f(x)$ that are in the corresponding interval. *Hint:* there are two.

Solution: Recall a critical number c is one where $f'(c) = 0$ or $f'(c)$ does not exist. Thus, we first compute the derivative to obtain:

$$f'(x) = 8x^{1/3} - x^{-2/3}.$$

There are a number of ways to proceed here to find the critical points. But one immediate observation we can make is that something weird happens when $x = 0$. To make the behavior of this more clear, we can factor:

$$f'(x) = x^{-2/3}(8x - 1) = \frac{8x - 1}{x^{2/3}}.$$

This factorization makes it easy to read off the critical numbers. At $x = 1/8$, $f'(1/8) = 0$ and at $x = 0$ $f'(0)$ does not exist. A number of you struggled to do the algebra here, but it is a simple manipulation of exponents. As mentioned above, this is not the only way to get this result.

3 (b) Determine the absolute extrema (min/max) of $f(x)$ in the interval.

Solution: We know that for a closed interval, we know that the min and max must occur at an endpoint or a critical number, so need only check the value of $f(x)$ at these points:

$$\begin{aligned} f(-1) &= 6(-1)^{4/3} - 3(-1)^{1/3} = 6 + 3 = 9. \\ f(0) &= 6(0)^{4/3} - 3(0)^{1/3} = 0. \\ f(1/8) &= 6(1/8)^{4/3} - 3(1/8)^{1/3} = 6 \cdot (1/16) - 3/8 = -9/8. \\ f(1) &= 6(1)^{4/3} - 3(1)^{1/3} = 6 - 3 = 3. \end{aligned}$$

Thus, we see the minimum occurs at $x = 1/8$ and the maximum occurs at $x = 1$.

2. Consider the following function:

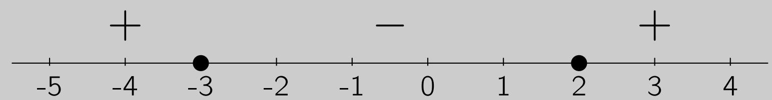
$$g(x) = 2x^3 + 3x^2 - 36x + 54.$$

3 (a) Use the **first derivative test** to identify the local extrema (min/max) of $g(x)$.

Solution: To use the first derivative test to identify the minima and maxima, we must first compute the first derivative:

$$g'(x) = 6x^2 + 12x - 36 = 6(x + 3)(x - 2).$$

Thus, we see that the critical numbers are $x = 2, x = -3$. By testing values, we can determine the sign of the derivative which can be summarized by the number line below:



$g'(x)$

By the first derivative test, since the the function increases and then decreases at $x = -3$, we know that it must be a local max. Similarly, at $x = 2$, since the function decreases and then increases, we must have a local min.

One very important distinction to make is between the first problem and this problem. Notice, for the first problem, we were finding **global** extrema on an **interval**. Here, we are just finding **local** extrema.

2 (b) Use the **second derivative test** to confirm your answer to part (a).

Solution:

We first compute the second derivative:

$$g''(x) = 12x + 6, \tag{1}$$

At our two critical values, we have that $g''(-3) = -30$, meaning at the critical point, the function is concave down (like a frown), and therefore a local maximum. Similarly, at $g''(2) = 30$, the function is therefore concave up (like a cup), and a local minimum. Thankfully, this agrees with our answer from part (a).