

Name: _____

Quiz Score: _____/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

1. For this problem, consider the following limit:

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x^2 + x - 30}$$

3 (a) Consider the following approach:

$$\frac{x^2 - 25}{x^2 + x - 30} = \frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x+6)} = \frac{x+5}{x+6}$$

Why is this **incorrect**? Hint: Are $\frac{x^2-25}{x^2+x-30}$ and $\frac{x+5}{x+6}$ the same function? Consider $x = 5$.

3 (b) Compute the limit correctly and explain why this differs from the reasoning of part (a).

Solution: Hopefully I emphasized this enough in class, but there is a fatal flaw with the approach described in (a), although it is a subtle one. By just writing the equation as is, as done in (a) and canceling out $x - 5$ from the top and bottom, one is ignoring the fact that $x - 5 = 0$ when $x = 5$. **It does not make sense to cancel out $\frac{0}{0}$.**

The key distinction between the approach in (a) and the correct approach is that **in the limit, we do not actually evaluate at $x = 5$.** Thus, we can write:

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x^2 + x - 30} = \lim_{x \rightarrow 5} \frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x+6)} = \lim_{x \rightarrow 5} \frac{x+5}{x+6} = \frac{10}{11}$$

Note the important difference: we are allowed to cancel $x - 5$ in the limit for the reason listed above. This is subtle but extremely important in developing a deeper understanding of limits and how to calculate them.

I tried to be as generous as possible grading this problem, giving half the points or more if you wrote anything coherent down. In the future, **I will continue to penalize this mistake**, but not majorly because I too sometimes get "sloppy" and forget to write the limit. Regardless, you need to have an understanding why these two cases are different.

- 4 2. Compute the following limit if it exists, and if it does not, state why:

$$\lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h}.$$

Solution: Using our standard trick of multiplying by the conjugate:

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h} &= \lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h} \cdot \underbrace{\frac{\sqrt{1+h} + 1}{\sqrt{1+h} + 1}}_{=1} \\ &= \lim_{h \rightarrow 0} \frac{1 + h - 1}{h(\sqrt{1+h} + 1)} \\ &= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{1+h} + 1)} \\ &= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{1+h} + 1)} \\ &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{1+h} + 1} \\ &= \frac{1}{\sqrt{1+0} + 1} \\ &= \frac{1}{2} \end{aligned}$$

A number of you tried to evaluate the limit too early in the algebra. Note that we basically wait as long as possible to do so, as in the above calculation.