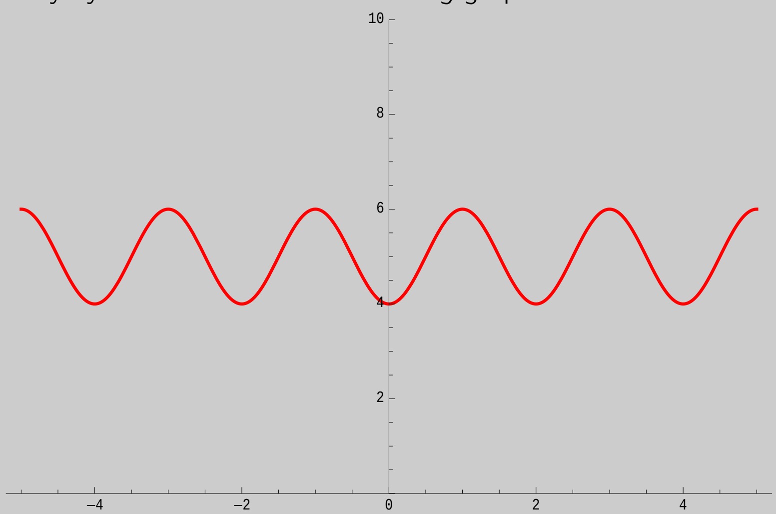
Name:____________________ Quiz Score:____/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

1. For this problem, let $f(x) = \cos(x)$ and $g(x) = e^x - 1$.

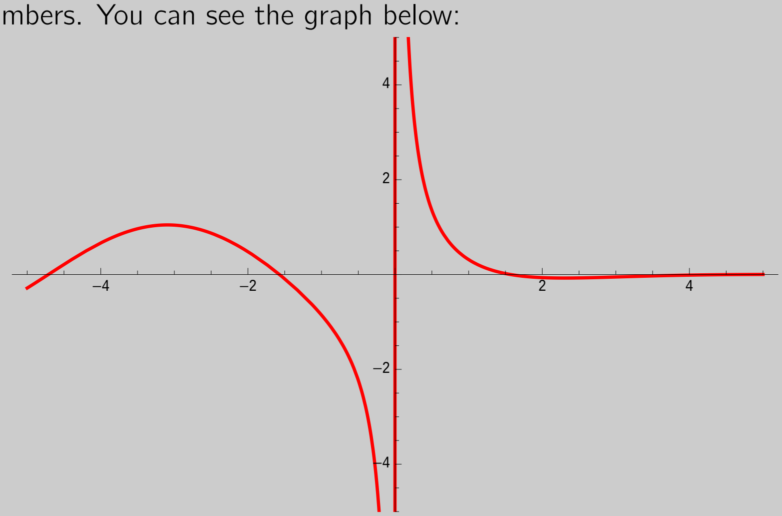
[1] (a) What is $-f(\pi x) + 5$? Plot it below.

Solution: $-f(\pi x) + 5$ is $f(x)$ shifted upward by 5, flipped over the y-axis (due to the minus sign) and scaled horizontally by a factor of π. The resulting graph is:



[2] (b) What is $\left(\frac{f}{g}\right)(x)$? What is its domain?

Solution: $\left(\frac{f}{g}\right)(x)$ can be computed simply by dividing the two functions, yielding: $\left(\frac{f}{g}\right)(x) = \frac{\cos(x)}{e^x-1}$. Note, the only point in the domain that is a problem is when $e^x - 1 = 0 \implies e^x = 1 \implies x = 0$. Otherwise, all points are good, meaning the domain is $\{x \in \mathbb{R} : x \neq 0\}$. The range in this case is all real numbers. You can see the graph below:



[1] (c) What is $f \circ g$? $g \circ f$?

Solution: $f \circ g = f(g(x)) = \cos(e^x - 1)$. $g \circ f = g(f(x)) = e^{\cos(x)} - 1$. There really are no ways to simplify these forms.

3 2. For this problem, consider $f(x) = \ln[2 + \ln(x)]$. What is f^{-1} and what is its domain?

Solution: Recall our technique for solving for the inverse. We start with $y = \ln[2 + \ln(x)]$ and attempt to solve for x:

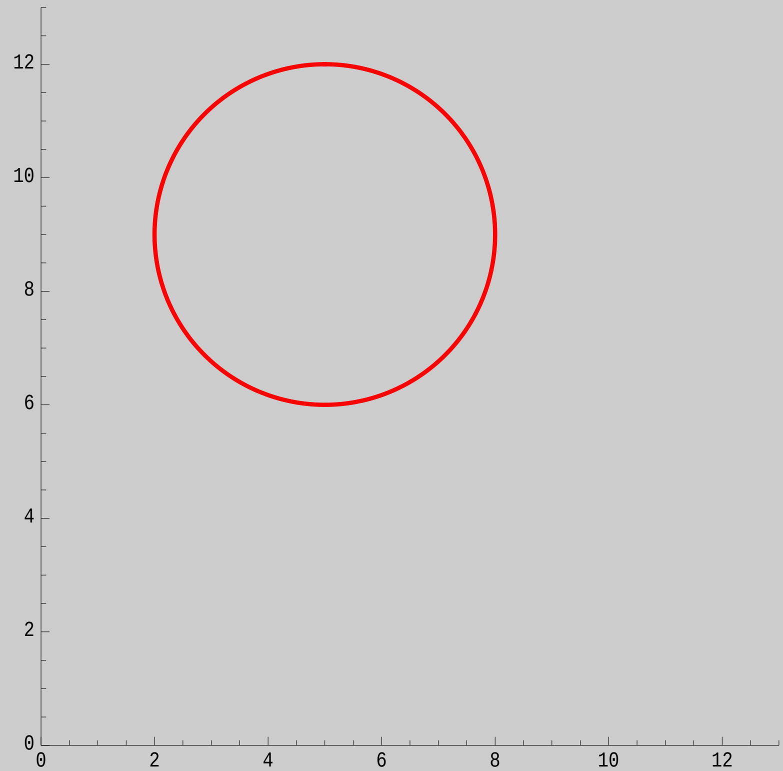
$$\begin{aligned} y &= \ln[2 + \ln(x)] \\ e^y &= 2 + \ln(x) \quad \text{(exponentiating both sides)} \\ e^y - 2 &= \ln(x) \quad \text{(moving -2 term)} \\ e^{e^y - 2} &= x \quad \text{(exponentiating both sides again)} \\ \frac{e^{e^y}}{e^2} &= x \implies f^{-1}(x) = \frac{e^{e^x}}{e^2} \end{aligned}$$

Note, there are no problems with exponentials as far as domain. Also, e^2 is just a number, meaning it cannot cause a problem in the denominator. This means the domain of the inverse is $\mathbb{R}$, or $(-\infty, \infty)$.

3 3. Using the ideas of parametric curves, plot the following:

$$x(t) = 3\sin(t) + 5, \qquad y(t) = 3\cos(t) + 9, \qquad 0 \leq t \leq 2\pi$$

Solution: We know the famous trig identity $\cos(x)^2 + \sin(x)^2 = 1$, but notice that in this problem, we have a factor of 3 in front of the sin and cos term. We can deal with this by multiplying the trig identity on both sides by 9, yielding: $[3\cos(x)]^2 + [3\sin(x)]^2 = 9$, but also note that $3\cos(x) = y - 9$, $3\sin(x) = x - 5$, meaning we really have $(y-9)^2 + (x-5)^2 = 3^2$, which is just the equation of a circle centered at $(5, 9)$ with radius 3, which can be seen below:



I noticed a number of students plotted two separate curves, but recall the idea of parametric equations: there is a single parameter t given for a single curve. A single time t, say $t = 5$ corresponds to a single point on the curve whose x value is $x(5)$ and y value is $y(5)$.