HW7 Solutions

3.5] 4, 10, 34
3.6] 6, 10, 20, 40
3.7] 14, 32, 40.

3.5.4] $2\sqrt{x} + \sqrt{y} = 3$

$$2x^{1/2} + y^{1/2} = 3$$

Derivative:

$$x^{-1/2} + \frac{1}{2} y^{-1/2} \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^{-1/2}}{\frac{1}{2} y^{-1/2}}.$$

3.5.10] $1 + x = \sin(xy^2)$

Derivative:

$$1 = \cos(x \cdot y) \cdot \frac{d}{dx}(x \cdot y^2).$$

By product rule:

$$\frac{d}{dx}(x \cdot y^2) = 1 \cdot y^2 + x \cdot 2y \cdot \frac{dy}{dx}$$

Thus:

$$1 = \cos(xy^2)\left[1 \cdot y^2 + x\,2y \cdot \frac{dy}{dx}\right]$$

$$1 - \cos(xy^2) \cdot y^2 = 2xy\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1-\cos(xy^2) \cdot y^2}{2xy}.$$

3.5.34] $x^4 + y^4 = a^4$.

Here, a is constant.

Derivative:

$$4x^3 + 4y^3 \cdot y' = 0.$$

$$y' = -\frac{x^3}{y^3}$$

$$y'' = \frac{d}{dx}\left(\frac{-x^3}{y^3}\right) \Rightarrow \text{quotient rule!}$$

$$y'' = \frac{(-3x^2)\cdot y^3 - (3y^2\cdot y')\cdot(-x^3)}{y^3}.$$

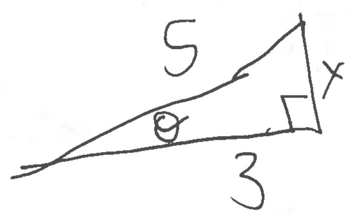
We know $y' = \frac{-x^3}{y^3}$. Plug it in.

Therefore

$$y'' = \frac{(-3x^2)\cdot y^3 - (3y^2)\left(\frac{-x^3}{y^3}\right)\cdot(-x^3)}{y^3}.$$

$\boxed{3.6.6}$ $\csc\left(\arccos\frac{3}{5}\right).$

$\arccos\frac{3}{5}$ gives you θ in the following triangle.

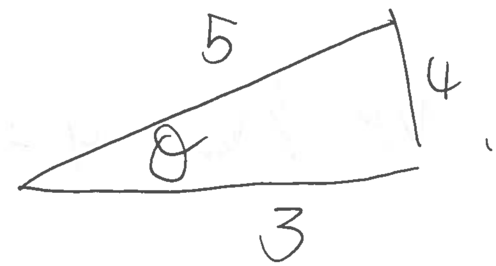


since $\cos = \frac{\text{adjacent}}{\text{hypotense}}$.

$\therefore x = \sqrt{5^2 - 3^2}$
$= \sqrt{25-9} = 4.$

Thus, we have

csc(θ) where



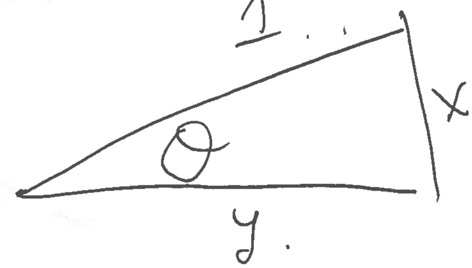
$\csc(\theta) = \frac{1}{\sin(\theta)}$. $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$.

$= \frac{4}{5}$.

$$\boxed{\csc(\theta) = \frac{5}{4}}$$

$\boxed{3.6.10}$ $\tan(\sin^{-1} x))$. Similar to before without numbers!

$\sin^{-1}\left(\frac{x}{1}\right) = \theta$:



we know $y = \sqrt{1^2 - x^2}$. by Pythag. theorem.

Thus, $\tan(\theta) = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{x}{y} = \dfrac{x}{\sqrt{1-x^2}}$.

3.6.20] $F(\theta) = \arcsin\left(\sqrt{\sin\theta}\right)$.

$$F = \arcsin\left(\sqrt{\sin\theta}\right).$$

apply sin to both sides: $\sin(F) = \sqrt{\sin\theta}$.

derivative:

$$\underbrace{\cos(F)\cdot\frac{dF}{d\theta}}_{\text{by chain rule/implicit diff}} = \frac{1}{2}\underbrace{(\sin\theta)^{-3/2}\cdot\cos\theta}_{\text{chain rule}}.$$

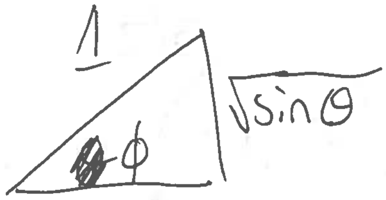
$$\frac{dF}{d\theta} = \frac{\frac{1}{2}(\sin\theta)^{-3/2}\cdot\cos\theta}{\cos(F)}.$$

but we know

$$F = \arcsin\left(\sqrt{\sin\theta}\right)$$

$\therefore$

$$\frac{dF}{d\theta} = \frac{\frac{1}{2}(\sin\theta)^{-3/2}\cdot\cos\theta}{\cos\left[\arcsin\left(\sqrt{\sin\theta}\right)\right]}$$



$$= \frac{\frac{1}{2}(\sin\theta)^{-3/2}\cdot\cos\theta}{\sqrt{1-\sin(x)}}$$

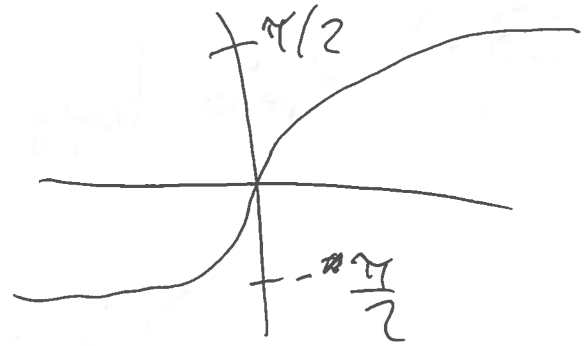
$\boxed{3.6.40}$ $\lim_{x\to 0^+} \tan^{-1}(\ln(x))$

$\ln(x)$:

$\therefore \lim_{x\to 0^+} \ln(x) = -\infty.$

$\therefore \lim_{x \to 0^+} \tan^{-1}(\ln(x)) = \lim_{y \to -\infty} \tan^{-1}(y)$.

$\tan^{-1}$:



$\therefore \lim_{y \to -\infty} \tan^{-1}(y) = \boxed{-\frac{\pi}{2}}$.

3.7.14) $F(y) = y\ln(1+e^y)$.

Product rule:

$$F' = \frac{d}{dy}(y) \cdot \ln(1+e^y) + y \cdot \frac{d}{dy}(\ln(1+e^y))$$

$$= \left[1 \cdot \ln(1+e^y)\right] + y \cdot \frac{\frac{d}{dy}(1+e^y)}{1+e^y} =$$

$$\boxed{F' = \ln(1+e^y) + y' \frac{e^y}{1+e^y}.}$$

3.7.32) $f(x) = \log_a(3x^2-2)$

$$f'(x) = \frac{1}{\ln(a)} \cdot \frac{\frac{d}{dx}(3x^2-2)}{3x^2-2}$$

$$= \frac{1}{\ln(a)} \cdot \frac{6x}{3x^2-2}.$$

$f'(1) = 3 \Rightarrow$

$$3 = \frac{1}{\ln(a)} \cdot \frac{6(1)}{3(1)^2-2}.$$

$$\ln(a)\, 3(1) = 6.$$

$$\ln(a) = 2.$$

$$\boxed{a = e^2.}$$

3.7.40) $y = \sqrt{x}^{x}$.

$\ln(y) = x \ln(\sqrt{x})$.

$= x \ln(x) \cdot \frac{1}{2}$

$= \frac{1}{2} x \ln(x)$.

Derivative:

product rule:

$\frac{y'}{y} = \frac{1}{2}\ln(x) + \cancel{\frac{1}{2}x} \cdot \cancel{\frac{1}{x}}$

$= \frac{1}{2}\ln(x) + \frac{1}{2}$

$= \frac{1}{2}(1 + \ln(x))$.

$y' = \frac{y}{2}(1 + \ln(x))$

$y = \sqrt{x}^{x} \Rightarrow$

$\boxed{y' = \sqrt{x}^{x}(1 + \ln(x))}$.