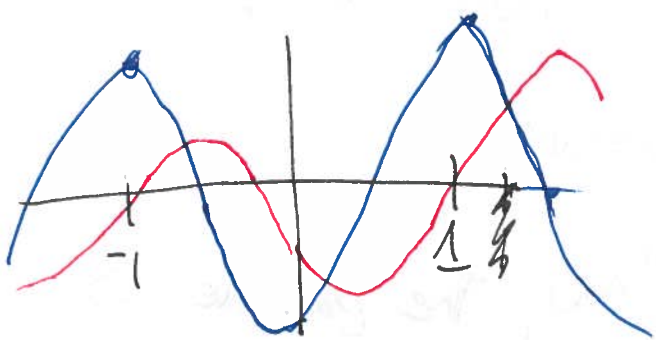
HW5 Solutions

2.8] 6, 12
3.1] 10, 26, 30, 22, 62
3.2] 10, 18, 22, 42.

2.8.6] I like this Problem because there are several layers to it. First, which graph is f? f'?



Notice at $x=1$, the blue graph has slope and the red function has a zero. Can we conclude blue$'$ = red? NO!

Notice to the right of $x=1$, blue is <u>decreasing</u>, but red > 0, thus blue$'$ ≠ red.

Thus, red$'$ = blue, but we can see this by examining the other peaks.

From this, we can see that blue $= f'$, meaning $f''(1)$ occurs at blue's peak and is 0. This also corresponds to an inflection point ~~on~~ in the red graph. $f'(-1)$ is some positive number and is therefore greater.

2.8.12) Whenever

$f' < 0$, the particle is moving left (getting closer to 0)

$f' > 0 \Rightarrow$ moving right.

Positive acceleration $\Rightarrow f'' > 0$

which is concave up "like a cup."

i.e. from $\boxed{[4, 6]}$

3.1.10) $h(x) = (x-2)(2x+3)$

~~We use the product rule:~~

$$\cancel{\frac{d}{dx}(h(x)) = \frac{d}{dx}\{(x-2)\}(2x+3) + (x-2)\cdot\frac{d}{dx}\{(2x+3)\}}$$

$$\cancel{= 1\cdot 2x+3 +}$$

Whoops. We don't use the product rule yet. That's section 3.2.

Expand: $(x-2)(2x+3) = 2x^2+3x-4x-6$

$$= 2x^2-x-6.$$

Use the power rule:

$$\frac{d}{dx}(2x^2-x-6) = 4x-1$$

Easier than product rule anyway.

3.1.28) $y = x^4+2x^2-x$ @ $(1,2)$.

$$\frac{d}{dx}(x^4+2x^2-x) = 4x^3+4x-1.$$

Thus, ~~as~~ ~~they~~ $f'(1)=7$

and the tangent line is:

$$y-2=7(x-1).$$

$\boxed{3.1.30}$ $y=(1+2x)^2$, @ $(1,9)$

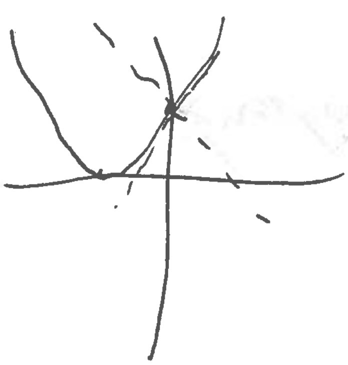
$$= 1+4x+4x^2.$$

$$\frac{d}{dx}(1+4x+4x^2) = 4+8x,$$

$f'(1)=12$ = slope of tangent line.

Tangent: ~~Slope~~ $y-9=12(x-1)$.

Normal line's slope: $-\frac{1}{f'(1)} = -\frac{1}{12}$.



Normal line:

$$y-9=-\frac{1}{12}(x-1).$$

Same point, different slope.

3.1.22) $y = ae^v + \frac{b}{v} + c/v^2$

$= ae^v + bv^{-1} + cv^{-2}$

$\frac{d}{dv}\left(ae^v + bv^{-1} + cv^{-2}\right)$

$= ae^v + (-1)bv^{-2} + (-2)cv^{-3}$

$= ae^v - \frac{b}{v^2} - \frac{2c}{v^3}$

3.1.62) I don't want to go too deeply into this problem but philosophically, I think it's neat. You'll see a ton of differential equations later so this is just a taste.

If we plug in the supposed solution

$y = Ax^2 + Bx + C$ to our DE:

$y'' + y' - 2y = x^2$

What we're left with is:

$$y'' = 2A$$

$$y' = 2Ax + B$$

$$y = Ax^2 + Bx + C,$$

$\Rightarrow$

$$y'' + y' - 2y = x^2 \Rightarrow$$

$$2A + 2Ax + B - 2(Ax^2 + Bx + C) = x^2.$$

The big idea here is to group powers of x.

$\Rightarrow$

$$(-2A)x^2 + (2A - 2B)x + 2A + B + C = x^2.$$

Notice that the coefficient of x^2, x, and constants must match on both sides, telling us:

$$-2A=1 \quad (x^2 \text{ coeff})$$
$$2A-2B=0 \quad (x \text{ coeff})$$
$$2A+B+C=0 \quad (\text{constant coeff})$$

$$A=\frac{-1}{2}.$$
$$B=\frac{1}{2}.$$
$$C=-\frac{1}{2}.$$

This method is called "undetermined" coefficients because all we had to do was determine A, B, C to solve the differential equation.

$\boxed{3.2.10}$

$$R(t)=(t+e^t)(3-\sqrt{t})$$

Product rule:

$$\frac{d}{dx}(R(t))=\frac{d}{dx}\{(t+e^t)\}(3-\sqrt{t})+\frac{d}{dx}\{(3-\sqrt{t})\}\cdot(t+e^t)$$

$$= (1+e^t)(3-\sqrt{t}) + (t+e^t)\left(-\frac{1}{2}t^{-1/2}\right)$$

3.2.18] $z = w^{3/2}(w+ce^w)$

$$\frac{dz}{dw} = \frac{d}{dw}(w^{3/2})\cdot(w+ce^w)$$

$$+ \frac{d}{dw}(w+ce^w)\cdot w^{3/2}$$

$$= \frac{3}{2}w^{1/2}(w+ce^w)$$

$$+ (1+ce^w)w^{3/2}$$

3.2.22) $f(x) = \dfrac{1-xe^x}{x+e^x} \begin{array}{l}\leftarrow g(x)\\ \leftarrow h(x)\end{array}$

$f'(x) = \dfrac{g'h - h'g}{h^2}$ $g' =$ by product rule

$-e^x - xe^x = -e^x(1+x)$

$$= \frac{-e^x(1+x) - (1+e^x)(1-xe^x)}{(x+e^x)^2} = \text{Something}$$

3.2.42)

a.) $h = 5f - 4g$

$h' = 5f' - 4g'$

$h'(2) = 5f'(2) - 4g'(2)$

$= 5 \cdot {-2} - 4 \cdot 7$

$= -10 - 28 = -38.$

b.) $h(x) = f(x)g(x)$

$h'(x) = f'(x)g + g'(x) \cdot f.$

$h'(2) = f'(2) \cdot g(2) + g'(2)f(2)$

$= -2 \cdot 4 + 7 \cdot -3$

$= -29.$

c.) $h(x) = \frac{f}{g}$ $\quad h' = \frac{f'g - g'f}{g^2}$

$h'(2) = \frac{f'(2)g(2) - g'(2)f(2)}{g(2)^2} = \frac{-2 \cdot 4 - 7(-3)}{16}$

$= \frac{13}{16}$

d. $h'(x) = \dfrac{\frac{d}{dx}\{g(x)\}\cdot(1+f(x)) - \frac{d}{dx}\{1+f(x)\}\cdot g(x)}{(1+f(x))^2}$

$h'(2) = \dfrac{g'(2)(1+f(2)) - f'(2)\cdot g(2)}{[1+f(2)]^2}$

$= \dfrac{7\cdot(1+-3) - (-2)(4)}{(1+-3)^2}$

$= \dfrac{-14+8}{4} = \dfrac{-3}{2}.$