HW4 Sols

2.4) 36, 42
2.6) 14, 20, 31, 46, 50
2.7) 20, 24, 34, 36, 44, 52

2.4.36) I showed a demo of this problem in class. For $x \neq 2, 3$, we have regular polynomials which are continuous, so it's clear we must turn our attention to these two points.

Particularly, we need: $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$.

@2:

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{x^2-4}{x-2} = \lim \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$$

$$= 4.$$

$$\lim_{x \to 2^+} f(x) = a(2)^2 - b(2) + 3.$$

@3: $\lim_{x \to 3^-} f(x) = a(3)^2 - b(3) + 3$

$\lim_{x \to 3^+} f(x) = 2(3) - a + b.$

Thus, we have two equations with two unknowns!

(1) $4 = 4a - 2b + 3$

(2) $9a - 3b + 3 = 6 - a + b$.

There are many ways to solve this but I'll show one:

Consider (2)

$$9a - 3b + 3 - 6 + a - b = 0.$$

$$10a - 4b - 3 = 0.$$

$$10a = 4b + 3$$

$$a = \frac{2}{5}b + \frac{3}{10} \quad (3)$$

Plug into (1)

$$4 = 4\left(\frac{2}{5}b + \frac{3}{10}\right) - 2b + 3$$

$$\Rightarrow b = \frac{1}{2}.$$

Plug into (3) $\Rightarrow a = \frac{1}{2}$.

2.4.42) We want to prove $\sqrt[3]{x}=1-x$ has a root on the interval $(0,1)$.

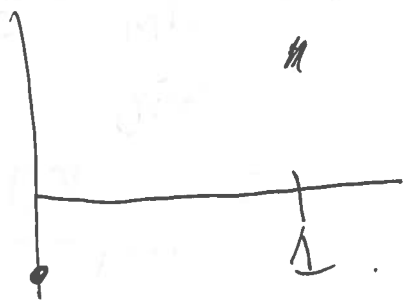
Rewriting, we want to prove there exists an x between 0 and 1 such that

$f(x)$ $\sqrt[3]{x}-1+x=0$.

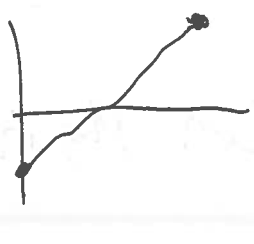
Note, this is a continuous function, meaning the IVP applies.

Consider @0: $f(0)=-1$.

Thus, we know



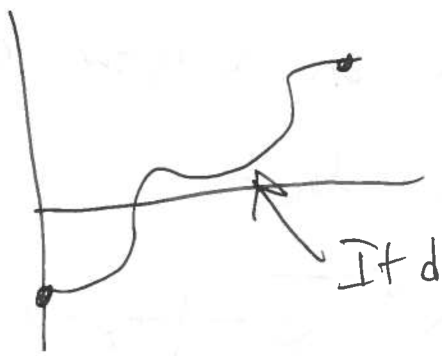
If we can prove $f(1)>1$, then there <u>must</u> be a root by the IVP



But what is

- $f(1)$?, $f(1)=1$,

Thus we have



It doesn't matter what the shape is, because it is continuous it **must** cross the axis, providing a root.

2.6.14) Height of rock $H(t) = 10t - 1.86t^2$.

$$\text{Velocity} = H'(t) = \lim_{h \to 0} \frac{H(t-h) - H(t)}{h}$$

$$= \lim \frac{10(t-h) - 1.86(t-h)^2 - 10t + 1.86}{h}$$

$$= 10 - 3.72t.$$

Can plug in $t=1$.

$H(t)$:

We want when it returns to surface, i.e. $H(a)=0$ above.

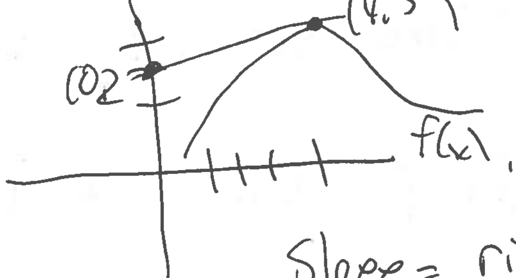
$H(t) = 10t - 1.86t^2 = t(10 - 1.86t)$

$t=0$ <u>or</u> $t = \frac{10}{1.86} = 5.37$.

↑ start

$H'(5.37)$ is ending velocity.

2.6.2) $y=f(x)$ @(4,3). passes thru (0,2).



$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{3-2}{4-0} = \frac{1}{4}$.

$= f'(4)$.

Tangent occurs at (4,3)

$\therefore f(4) = 3$.

2.6.31) $f'(a) = \lim_{h \to 0} \dfrac{f(a-h) - f(a)}{h}$

Conjugate.

$$= \lim \frac{\sqrt{1-2(a-h)} - \sqrt{1-2a}}{h} \cdot \frac{\sqrt{1-2(a-h)} + \sqrt{1-}}{\sqrt{1-2(a-h)} + \sqrt{}}$$

$$= \lim \frac{(\cancel{1} - \cancel{2(a}-h)) - \cancel{1} - \cancel{2a}}{h\left(\sqrt{1-2(a-h)} + \sqrt{1-2a}\right)}$$

$$= \lim \frac{-2\cancel{h}}{\cancel{h}\left(\sqrt{1-2(a-h)} + \sqrt{1-2a}\right)}$$

$$= \frac{-1}{\sqrt{1-2a}}$$

2.6.46). Since you all whined, I won't grade this problem but it's really not that bad.

Let $A = 100{,}000$

$B = -\frac{1}{60}$ for the sake of making the algebra nicer.

Then. $V(t) = A(1+Bt)^2$.

We want $V'(t)$:

$$V'(t) = \lim_{h \to 0} \frac{V(t-h) - V(t)}{h}$$

$$= \lim \frac{A[1+B(t-h)]^2 - A(1+Bt)^2}{h}$$

$$= A \lim \frac{[1+B(t-h)]^2 - (1+Bt)^2}{h}$$

$$= A \lim \frac{\cancel{1} - 2Bh + B^2h^2 \cancel{+2Bt} - 2B^2ht \cancel{+B^2t^2} - (\cancel{1} \cancel{+2Bt} \cancel{+B^2t^2})}{h}$$

$$= A \lim \frac{-2B\cancel{h} + B^2\cancel{h}^2 - 2B^2\cancel{h}t}{\cancel{h}}$$

$$= A \lim_{h \to 0} -2B + B^2h + 2B^2t.$$

$$= A(-2B + 2B^2t).$$

Thus, we can plug A, B back in:

$$V'(t) = 10000\left(+\frac{2}{60} + \frac{\cancel{\#}2t}{60^2}\right).$$

Okay. That did take a page of algebra. A bit annoying.

2.6.50) $Q = f(p)$ = quantity in pounds of Gourmet coffee for p dollars per pound.

This is basically an economics problem.

$f'(\cdot)$ describes the change in quantity sold <u>per</u> change of the price.

$f'(8)$ will very likely be negative.

If we increase the price, to say \$9th, since the price rose, fewer pounds will be sold.

2.7.20.) $y = mx + b$

$$f'(x) = \lim_{h \to 0} \frac{m(x+h) \cancel{+b} - [mx \cancel{+b}]}{h}$$

$$= \lim \frac{\cancel{mx} - \cancel{mx} + mh}{h}$$

$$= \lim m \frac{\cancel{h}}{\cancel{h}}$$

$$= m.$$

This is obvious since we are computing the slope of the line with slope m.

2.7.24) $f(x) = x + \sqrt{x}$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{(x+h)} + \sqrt{x+h} - [\cancel{(x)} + \sqrt{x}]}{h}$$

$$= \lim \frac{h + \sqrt{x+h} - \sqrt{x}}{h} = \lim \frac{h}{h} + \lim \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= 1 + \frac{1}{2\sqrt{x}}$$

2.7.34) Let $P(t)$ be percentage of Americans under 18.

$P'(t) =$ change (rate of) per year in percentage of Americans under 18.

$$P'(t) \approx \frac{P(t+10) - P(t)}{10}.$$

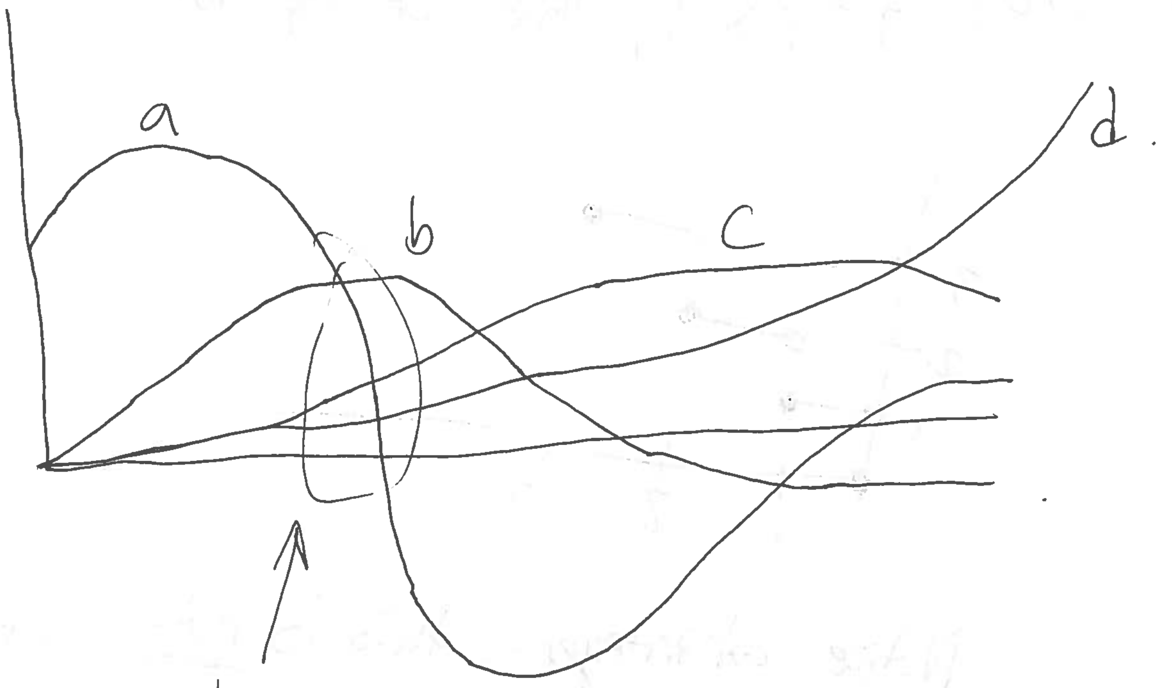
Can make a table using

$P'(t)$ estimate would be better if we had more data.

2.7.36). At $x=0$, f is not continuous, it is not differentiable.

At $x=3$, we have a corner, which is also not differentiable.

2.7.44).



Notice

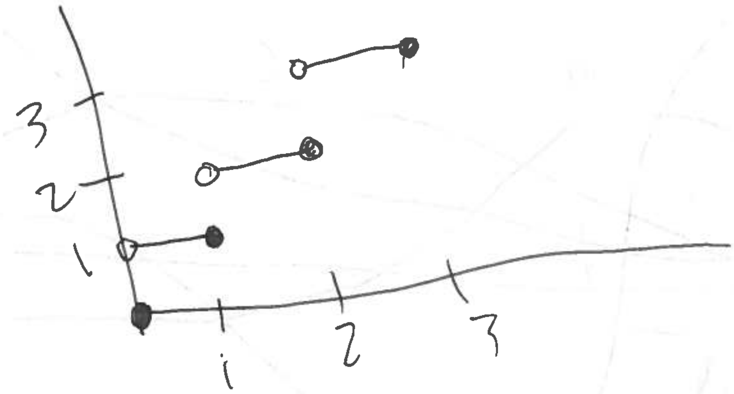
b is flat here, meaning its derivative is 0, thus $b' = c$.

The derivative of c must always stay ~~[illegible]~~ positive. except for at the end since c is increasing, thus ~~[illegible]~~ $c' = b$.

Finally, notice that d is also always increasing, thus $d' = c$.

thus: d position
c velocity
b acceleration
a jerk.

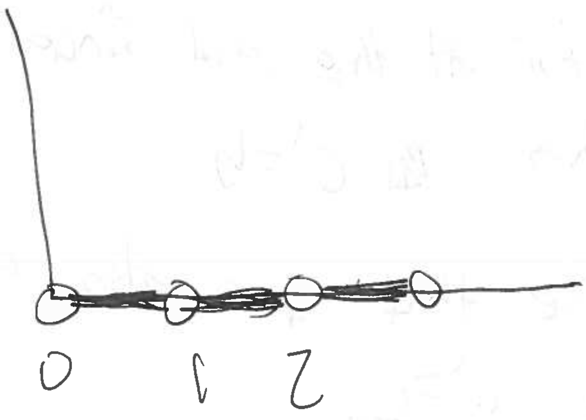
2.7.52) $y = \lceil x \rceil$, the ceiling function



Notice at integers, this is <u>not</u> diff'able.

In between, the derivative is zero since we have flat lines. Thus,

f' looks like:



$$f'(x) = \begin{cases} 0 & \text{if } x \neq \text{integer } (\mathbb{Z}). \\ \text{DNE} & \text{otherwise} \end{cases}$$