HW2 Solutions

2.2.6) (a.) $\lim_{x \to -3^-} h(x)$.

Note, even though it is a weird shape, $h(x)$ approaches 4 as $x \to -3$ from the left,

(b.) $\lim_{x \to 3^+} h(x) = 4$.

Same as above, except from the right

(c.) By (b) and (a) matching, the limit <u>must</u> exist and is defined to be

$$\lim_{x \to -3} h(x) = 4.$$

(d.) $h(-3)$ is undefined. No shaded circle.

(e.) $\lim_{x \to 0^-} h(x) = 1$.

(f.) $\lim_{x \to 0^+} h(x) = -1$

(g.) $\lim_{x \to 0} h(x)$ DNE because (e) · (f) differ.

h) $h(0) = 1$ by the shaded in circle.

(i.) $\lim_{x \to 2} h(x). = 2$

(j.) $h(2)$ DNE.

(l) $\lim_{x \to 5^-} h(x)$. DNE. Note, this is similar to our example in class.

swap.

(k) $\lim_{x \to 5^+} h(x)$. It appears as though it approaches 3 from the right. Thus, $\lim_{x \to 5^+} h(x) = 3$. despite the mess on the left side, which we can ignore when taking the right hand limit.

2.2.10) $f(x) = \dfrac{x^2+x}{\sqrt{x^3+x^2}}$.

$\lim_{x\to 0^-} f(x)$. This problem is harder than I originally thought.

First point. $x \to 0^-$ means <u>x is negative</u>, is that okay?

We need $x^3+x^2 \geq 0$ for $\sqrt{(\,\cdot\,)}$ to make sense. Therefore:

$$x^3+x^2 \geq 0.$$

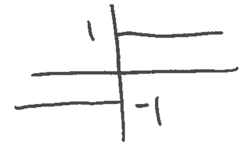
$$\underbrace{x^2}_{\text{always } \geq 0}\underbrace{(1+x)}_{1+x \geq 0 \Rightarrow x \geq -1.} \geq 0$$

Thus in the region around/near 0, we are okay.

But what is limit? Note.

$$\frac{x^2+x}{\sqrt{x^3+x^2}} = \frac{x(x+1)}{\sqrt{x^2(x+1)}} = \frac{x(x+1)}{|x|\sqrt{x+1}}.$$

$\dfrac{x}{|x|} = \operatorname{sgn}(x)$, meaning it gives you the sign of x. $\longrightarrow$



Thus we have

$$\lim_{x \to 0^-} \operatorname{sgn}(x) \cdot \frac{(x+1)}{\sqrt{x+1}}$$

Clearly $\dfrac{(x+1)}{\sqrt{x+1}} \to 1$

Thus we're left with $\lim_{x \to 0} \operatorname{sgn}(x) = -1$.

Similarly, $\lim_{x \to 0^+} f(x)$ boils down to

$$\lim_{x \to 0^+} \operatorname{sgn}(x) = 1.$$

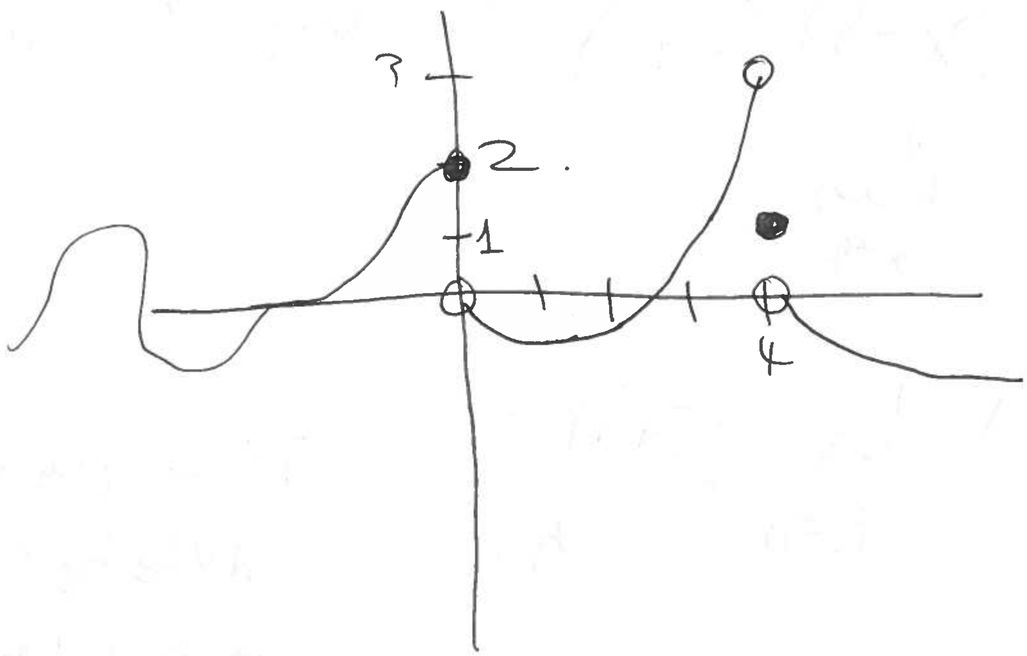
Thus $\lim_{x \to 0} f(x)$ DNE!

2.2.16) We want:

$\lim_{x \to 0^-} f(x) = 2$ $\lim_{x \to 0^+} f(x) = 0$ $\lim_{x \to 4^-} f(x) = 3$

$\lim_{x \to 4^+} f(x) = 0$, $f(0) = 2$, $f(4) = 1$.

One such example:



2.3.10) $\lim_{x \to 4} \frac{x^2 - 4x}{x^2 - 3x - 4}$. It's a ~~[illegible]~~ rational expression. What can we do? Plug 4 in!

$$\frac{(4)^2-4(4)}{(4)^2-3(4)-4} = \frac{0}{16-12-4} = \frac{0}{0} = ?$$

This is bad. We have to try something else!

factor:

$$\lim_{x\to 4} \frac{x\cancel{(x-4)}}{\cancel{(x-4)}(x+1)} = \lim_{x\to 4} \frac{x}{x+1} = \frac{4}{4+1} = \frac{4}{5}$$

because $x \neq 4$.

2.3.16) $\displaystyle\lim_{h\to 0} \frac{(2+h)^3-8}{h}$. If we plug in $h=0$, divide by 0, which is bad. What do we do?

Expand!

$$(2+h)^3-8 = 8+h^3+6h^2+12h-8$$

$$= h(h^2+6h+12).$$

Thus:

$$\lim_{h\to 0} \frac{(2+h)^3-8}{h} = \lim_{h\to 0} \frac{\cancel{h}(h^2+6h+12)}{\cancel{h}}$$

$$= \lim_{h\to 0} h^2+6h+12$$

$$=12.$$

2.3.24)

$$\lim_{x\to -4} \frac{\sqrt{x^2+9}-5}{x+4}.$$

Our standard trick: multiply by conjugate!

$$\lim_{x\to -4} \frac{\sqrt{x^2+9}-5}{x+4} \cdot \frac{\sqrt{x^2+9}+5}{\sqrt{x^2+9}+5}.$$

x^2+9-25
$=x^2-16$
$=(x+4)(x-4).$

$$= \lim_{x\to -4} \frac{x^2+9-5^2}{(x+4)(\sqrt{x^2+9}+5)}.$$

$$= \lim_{x\to -4} \frac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}(\sqrt{x^2+9}+5)}.$$

$$= \lim_{x\to -4} \frac{x-4}{\sqrt{x^2+9}+5} = \frac{-8}{10} = -\frac{4}{5}.$$

2.3.32) We want to prove that

$$\lim_{x \to 0^+} \sqrt{x}\, e^{\sin(\pi/x)} = 0,$$

This should <u>Scream</u> Squeeze Theorem.

Meaning we want $f(x), g(x)$:

$$f(x) \le \sqrt{x}\, e^{\sin(\pi/x)} \le g(x)$$

where $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} g(x) = 0$

Typically, we start by noticing that sin or cos are bounded;

$$-1 \le \sin \le 1.$$

but we have

$e^{\sin}$, resulting in:

$$e^{-1} \leq e^{\sin} \leq e^{1}.$$

But we want $\sqrt{x}\, e^{\sin}$,

so multiply by $\sqrt{x}$:

$$\underbrace{\frac{\sqrt{x}}{e}}_{g(x)} \leq \underbrace{\sqrt{x}\, e^{\sin}}_{\text{our } f(x)} \leq \underbrace{e\sqrt{x}}_{h(x)}.$$

Notice, $\lim_{x \to 0^+} g(x) = 0$

$\lim_{x \to 0^+} h(x) = 0.$

Thus, the limit of f is squeezed between these two, yielding:

$\lim_{x \to 0^+} f(x) = 0$ as desired.