# HW12 Solutions

5.5: 10, 26, 48
5.6: 4, 12, 16
5.7: 22, 28
5.10: 2, 20, 28, 43

5.5.10) $\int (3t+2)^{2.4}\,dt$

$u = 3t+2$

$du = 3\,dt$

$dt = \frac{du}{3}$

$$\rightarrow \int (u)^{2.4}\,\frac{du}{3}$$

$$= \frac{1}{3}\int u^{2.4}\,du$$

$$= \frac{1}{3}\,\frac{u^{3.4}}{3.4} + C$$

$$= \frac{(3t+2)^{3.4}}{3(3.4)} + C$$

5.5.26) $\int \frac{\cos\left(\frac{\pi}{x}\right)}{x^2}\,dx$

$u = \frac{\pi}{x}$

$du = -\frac{\pi}{x^2}\,dx$

$-\frac{du}{\pi} = \frac{dx}{x^2}$

$$-\int \frac{\cos(u)\, du}{\pi} = -\frac{1}{\pi}\int \cos(u)\, du$$

$$= -\frac{1}{\pi}\sin(u) + C$$

$$= -\frac{1}{\pi}\sin\left(\frac{x}{\pi}\right) + C$$

5.5.48) $\displaystyle\int_0^{\pi/2} \cos x \sin(\sin x)\, dx.$

$u = \sin x$

$du = \cos x\, dx.$

$x = 0,\ u = 0$

$x = \pi/2,\ u = 1$

$$= \int_0^1 \sin(u)\, du$$

$$= -\cos(u)\Big|_{u=0}^{u=1} = -\cos(1) + 1.$$

5.6.4) $\int xe^{-x}\,dx$

$u = x \qquad du = dx$

$dv = e^{-x}\,dx \qquad v = -e^{-x}$

$\int u\,dv = vu - \int v\,du.$

$= -e^{-x}\cdot x - \int(-e^{-x})\,dx$

$= -xe^{-x} + \int e^{-x}\,dx$

$= -xe^{x} - e^{-x} + C.$

5.6.12) $\int \sin^{-1} x\,dx$

$u = \sin^{-1} x \qquad du = \dfrac{dx}{\sqrt{1-x^2}}$

$dv = dx \qquad v = x$

$\int u\,dv = vu - \int v\,du$

$= x\sin^{-1} x - \int \dfrac{x}{\sqrt{1-x^2}}\,dx$

How do we evaluate $\int \frac{x}{\sqrt{1-x^2}}\,dx$?

Let $u = \sqrt{1-x^2}$

$$du = \frac{2x \cdot \frac{1}{2}}{\sqrt{1-x^2}}$$

$$= \frac{x}{\sqrt{1-x^2}}.$$

$$\int du = u + C$$

$$= \sqrt{1-x^2} + C.$$

5.6.16)

$$\int_0^1 (x^2+1)e^{-x} = uv\Big|_{x=0}^{x=1} - \int_0^1 v\,du$$

$u = (x^2+1)$ $du = 2x\,dx$

$dv = e^{-x}$ $v = -e^{-x}$

$$= (x^2+1)(-e^{-x})\Big|_{x=0}^{x=1} + 2\int_0^1 e^{-x}x\,dx$$

$$(-2e+1) + 2\int_0^1 e^{-x}x\,dx$$

$u = x \quad du = dx$

$dv = e^{-x}dx \quad v = -e^{-x}$

$$= -2e+1 + 2\left[-xe^{-x}\Big|_{x=0}^{x=1} + \int_0^1 e^{-x}dx\right]$$

$$= -2e+1 - 2e + e^{-1} - 1$$

5.7.22)

$$\int_0^1 \frac{x-4}{x^2-5x+6}dx = \int_0^1 \frac{x-4}{(x-3)(x-2)}dx$$

$$\frac{x-4}{x^2-5x+6} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$= \frac{A(x-3)+B(x-2)}{x^2-5x+6}.$$

Thus: $x-4 = A(x-3)+B(x-2)$.

Powers of $x$:

$x$: $1 = A+B$

Const: $-4 = -3A-2B$.

Solution: $B=-1$, $A=2$

$$\int_0^1 = \int_0^1 \left[\frac{2}{x-2} - \frac{1}{x-3}\right] dx.$$

$$= \ln|x-2| - \ln|x-3| \Big|_{x=0}^{x=1}$$

$$= \ln \underset{1}{\cancel{1}} - \ln 2 - (\ln(2) - \ln(3))$$

$$0 \quad = \cancel{\ln} - 2\ln 2 - \ln 3$$

5.7.28) $\displaystyle\int \frac{x^2 - x + 6}{x^3 + 3x}\,dx$

$$\frac{x^2 - x + 6}{x(x^2+3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3}$$

$$= \frac{A(x^2+3) + x(Bx + C)}{x(x^2+3)}$$

Thus: $x^2 - x + 6 = A(x^2+3) + x(Bx + C)$

Powers of $x$:

$x^2$: $1 = A + B$

$x$: $-1 = C$

Const: $6 = 3A$

---

Solution: $A = 2,\ B = -1,\ C = -1$

Thus:

$$\int = \int \left( \frac{2}{x} - \frac{x+1}{x^2+3} \right) dx$$

$$= \int \left[ \frac{2}{x} - \frac{x}{x^2+3} - \frac{1}{x^2+3} \right] dx$$

$$= 2\ln|x| - \frac{1}{2}\ln|x^2+3| - \frac{1}{\sqrt{3}} \arctan\left(\frac{x}{\sqrt{3}}\right) + C$$

from u-sub $u = x^2 + 3$.

since $\int \frac{1}{a^2+x^2} = \frac{1}{a} \operatorname{atan}\left(\frac{x}{a}\right) + C$

5.10.2)

a.) $\int_1^2 \frac{1}{2x-1}\,dx$ discont @

$2x-1=0$

$x=\frac{1}{2}$.

Not in range we are integrating, so OK.

b.) $\int_0^1 \frac{1}{2x-1}\,dx$, Same as above except we do have a discont.

Must split:

$$\int_0^1 = \int_0^{1/2} + \int_{1/2}^1 \dots$$

c.) $\int_{-\infty}^{\infty} \frac{\sin}{1+x^2}\,dx$. Obviously improper since

$\int_{-\infty}^{\infty}$.

d. $\int_1^2 \ln(x-1)\,dx$. Issues @

$x-1=0$

$x=\underline{1}$.

Improper but Ok since:

$$\int_1^2 \ln(x-1) = \lim_{z\to 1^+} \int_z^2 \ln(x-1)\,dx.$$

5.10.20). $\int_{-\infty}^{\infty} x^3 e^{-x^4}\,dx = \int_0^{\infty} x^3 e^{-x^4}\,dx$

$+ \int_{-\infty}^{0} x^3 e^{-x^4}\,dx.$

If one of these converges, the other is basically the same.

$$\int_0^{\infty} x^3 e^{-x^4}\,dx. = \lim_{z\to\infty} \int_0^z x^3 e^{-x^4}\,dx$$

$u = -x^4$

$du = -4x^3\,dx$, $\quad x = \infty,\ u = -\infty$

$\quad x = 0,\ u = 0$

$-\frac{du}{4} = x^3\,dx$

---

$$= \lim_{z \to -\infty} \int \frac{e^u}{4}\,du$$

$$= \frac{e^u}{4}\Big|_{u=0}^{z}$$

$$= \lim_{z \to -\infty} \frac{e^z - 1}{4}$$

$$= -\frac{1}{4}$$

Similarly: $\int_{-\infty}^{0} = -\frac{1}{4}$. Thus total int $= \underline{0}$

Odd function

5.10.28) $\int_6^8 \frac{4}{(x-6)^4}\,dx$

$$\int_6^8 \frac{4}{(x-6)^3}\,dx = \lim_{z\to 6} \int_z^8 \frac{4}{(x-6)^3}\,dx$$

$$\boxed{\begin{aligned} u &= x-6 \\ du &= dx. \end{aligned}}$$

$u=8=$

$$= \lim_{z\to 6} 4\int_z^8 \frac{du}{u^3}\,dx.$$

$$= \lim_{z\to 6} \frac{4\cdot u^{-2}}{-2}\Bigg|_{x=z}^{x=8}$$

$$= \lim_{z\to 6} \frac{-2}{(x-6)^2}\Bigg|_{x=z}^{x=8}$$

$$= \lim_{z\to 6}\left(\frac{-2}{(2)^2} + \frac{2}{(z-6)^2}\right)$$

$0$

$$= -\frac{1}{2}.$$

Converges

5.10.43) $\int_0^\infty \frac{x}{x^3+1}\,dx.$

We must find something larger or smaller than this to compare.

This problem is harder than I thought.

Because we only know about $\int_1^\infty x^p\,dx$

not $\int_0^\infty x^p\,dx$

Thus, let's split this up:

$$\int_0^\infty \frac{x}{x^3+1} = \int_0^1 \frac{x}{x^3+1} + \int_1^\infty \frac{x}{x^3+1}.$$

In $[0,1]$ $x \geq x^4$.

$x^3+1 \geq x^3$.

Thus:

$$\frac{x}{x^3+1} \geq \frac{x^4}{x^3+1} \geq \frac{x^4}{x^3} = x.$$

$\int_0^1 x\,dx$ converges, so the first part is good.

This does not hold for $1 \to \infty$.

But here, we still have:

$$x^3+1 \geq x^3, \text{ thus;}$$

$$\frac{x}{x^3+1} \geq \frac{x}{x^3} = \frac{1}{x^2}.$$

And we know

$$\int_1^\infty \frac{1}{x^2}\,dx \quad \underline{\text{converges}};$$

so both integrals converge meaning the whole integral converges.