Homework 10 Solutions

4.5] 6, 12, 30, 34, 40, 42

4.6] 11, 22, 32, 44

4.7] 6, 14

4.8] 12, 26, 34, 36

4.5.6] $\lim\limits_{x\to 1} \frac{x^a-1}{x^b-1}$.

Notice we have $\frac{0}{0}$.

Applying L'Hôpital's.

$$\lim \frac{x^a-1}{x^b-1} = \lim \frac{\frac{d}{dt}(x^a-1)}{\frac{d}{dx}(x^b-1)} = \lim_{x\to 1} \frac{ax^{a-1}-1}{bx^{b-1}\ 1}$$

$$= \frac{a-1}{b-1}$$

4.5.12] $\lim\limits_{\theta\to\pi/2} \frac{1-\sin\theta}{\csc\theta}$

~~[illegible]~~

I think this problem has a typo.

Notice

$$\frac{1-\sin\left(\frac{\pi}{2}\right)}{\csc\left(\frac{\pi}{2}\right)} = \frac{0}{1} = 0.$$

<u>Not</u> indeterminant.

4.6.30] $\lim_{x\to 0^+} \sin x \ln x \to 0\cdot(-\infty) = ?$

$$= \lim \frac{\ln x}{\frac{1}{\sin x}}$$

$$= \lim \frac{\frac{d}{dx}[\ln x)}{\frac{d}{dx}\left[(\sin x)^{-1}\right]}$$

$$= \lim \frac{\frac{1}{x}}{-(\sin x)^{-2}\cdot\cos x}$$

$$= \lim \frac{-\sin^2 x}{x\cos x} \to \frac{0}{0}.$$ Still no good.

L`Hôpital's again:

$$= \lim \frac{-2\sin x \cdot \cos x}{\cos x - x\sin x} \to \frac{0}{1+0} = \boxed{0}$$

4.5.34] $\lim x \to 0 \quad \csc x - \cot x.$

$\infty - \infty = ?$

$$= \lim_{x \to 0} \frac{1}{\sin x} - \frac{1}{\tan x}$$

$$= \lim \frac{1}{\sin x} - \frac{\cos x}{\sin x}$$

$$= \lim \frac{1-\cos x}{\sin x} \to \frac{0}{0} = ?$$

L`Hôpital's: $$= \lim_{x \to 1} \frac{\sin x}{\cos x} = \frac{0}{1} = \boxed{0}.$$

4.5.40] $\lim_{x \to 0^+} (\tan 2x)^x$

$0^0 \to ?$

$\lim (\tan 2x)^x = e^{x \ln \tan 2x}$

$\swarrow$

$\lim_{x \to 0^+} x \ln \tan 2x = 0 \cdot (-\infty) = ?$

$= \lim_{x \to 0^+} \frac{\ln \tan 2x}{1/x}$

L'Hôpitals: $\lim_{x \to 0^+} \frac{2\sec^2(2x)/\tan(2x)}{-x^{-2}}$

$= \lim \frac{-x^2 2\sec^2(2x)}{\tan 2x}$

L'Hôpitals again:

$= \lim \frac{-2x \cdot 2\sec^2(2x) + (-x)^2 8 \tan(2x)\sec^2(2x)}{\sec^2 x}$

$= \frac{0+0}{1} \Rightarrow$ our limit is $e^0 = ①$.

4.5.42] $\lim_{x \to \infty} \left(1+\frac{a}{x}\right)^{bx}$,

$1^{\infty} \Rightarrow$,

$= \lim e^{\underline{bx \ln\left(1+\frac{a}{x}\right)}}$

$\infty \cdot 0 \Rightarrow$,

$\lim bx \ln\left(1+\frac{a}{x}\right) = \lim \frac{\ln\left(1+\frac{a}{x}\right)}{1/bx}$

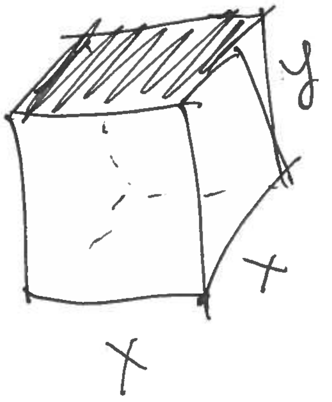
L'Hopital's

$$= \lim_{x \to \infty} \frac{\frac{-a}{ax+x^2}}{-\frac{1}{bx^2}} = \lim_{x \to \infty} \frac{abx^2}{x^2+ax}$$

$\rightarrow \boxed{ab}$,

Thus, total lim $\rightarrow \boxed{e^{ab}}$

$\boxed{4.6.11}$



$V = x^2 \cdot y = 32000$ ← constraint

$SA = x^2 + 2xy + 2xy$

$= x^2 + 4xy$ ← objective.

$0 \le x \le 32000$.

$y = \frac{32000}{x^2}$.

$SA = x^2 + 2x \cdot \frac{32000}{x^2} + \frac{2x \cdot 32000}{x^2}$ ..

$= x^2 + \frac{128000}{x}$.

$SA' = 2x - \frac{128000}{x^2} = 0$

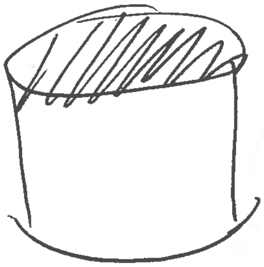
1st deriv test:

$2x^3 - 128000 = 0$

$x = \sqrt[3]{\frac{128000}{2}}$.

− +

4.6.22.]



just a number

$$\overset{\swarrow}{V} = \pi r^2 h \rightarrow \text{constant}$$

$$SA = 2\pi r \cdot h + \underset{\substack{\uparrow \\ \text{only one base}}}{\pi r^2} \Leftarrow \text{objective}$$

$$h = \frac{V}{\pi r^2}$$

$$SA = 2\pi r \cdot \frac{V}{\pi r^2} + \pi r^2$$

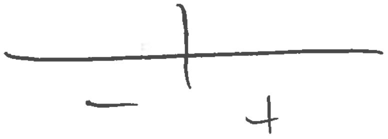
$$= \frac{2V}{r} + \pi r^2$$

$$SA' = 0 = \frac{-2V}{r^2} + 2\pi r$$

$$-2V + 2\pi r^3 = 0$$

$$r = \sqrt[3]{\frac{V}{\pi}}$$

Again 1st deriv test:



4.6.32]

$$E(v) = av^3 \cdot \frac{L}{v-u}.$$

$$= aL \cdot \frac{v^3}{v-u}.$$

By quotient rule:

$$E'(v) = aL\left[\frac{3v^2(v-u) - v^3(1)}{(v-u)^2}.\right]$$

Obviously $E'(v) = 0$ @ $v = 0$

and $E'(v)$ @ $v = u$ undefined.

One more critical #:

$$E'(v) = 0 \Rightarrow aL\, v^2[3(v-u) - v] = 0$$

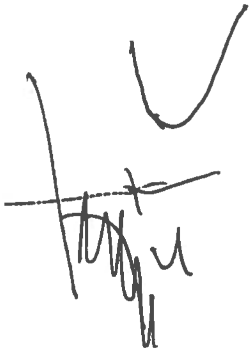
$$[3v - 3u - v] = 0$$

$$2v - 3u = 0$$

$$v = \frac{3u}{2}.$$

By the wording of the problem, I think $v > u$, meaning $v \neq 0$ and $v \neq u$,

Plot:



eliminating the first two critical #'s.

By 1st deriv test: $$\overset{-}{\rule{1cm}{0.4pt}}\underset{\frac{3u}{2}}{|}\overset{+}{\rule{1cm}{0.4pt}}\ v$$

Thus we have a min. Could also consider E''.

4.6.44] $$\underset{\substack{\uparrow\\ \text{Profit for selling } x \text{ items}}}{P(x)} = \underset{\substack{\uparrow\\ \text{revenue}}}{R(x)} - \underset{\substack{\uparrow\\ \text{cost}}}{C(x)}$$

To maximize P, we need critical #'s, so

$$P'(x) = 0 = R'(x) - C'(x)$$

$$\underset{\substack{\uparrow\\ \text{revenue of selling 1 more item if we are currently selling } x}}{R'(x)} = C'(x).$$

Same w/ $C'(x)$.

$R(x) = x \cdot p(x)$

$\therefore \quad P(x) = x\,p(x) - C(x)$.

Take derivative.

Overall, don't worry about this problem.

4.7.6] I didn't do this by hand.

$f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + 3 = 0, \quad x_0 = -3$

$f'(x) = x^2 + 2x = 0$.

See code.

```
oldx = -3;                             % sets our starting value
f=@(x) (1/3)*x^3+(1/2)*x^2+3;    % defines f
fprime=@(x) x^2+2*x;            % defines f'
n=0;                              % start on n=0
nmax=5;                           % max number of steps
tol=.00001;                       % tolerance to stop at;
difference = 1;                   % we will update this variable to be newx-x. start with 1.
xvals = [oldx];                   % store our x values in an array

while  tol<=difference&n<=nmax
    newx = oldx - f(oldx)/fprime(oldx);  % compute new x based on our formula
    xvals = [xvals; newx];                % add new x value to our array
    difference = abs(newx-oldx);         % compute difference to see if we are close
    oldx = newx;                         % continue iterating
    n=n+1;
end

disp(xvals);  %print out xvalues


>> newtonsmethod
   -3.0000
   -2.5000
   -3.2333
   -2.4709
   -3.3510
   -2.4833
   -3.2990
```

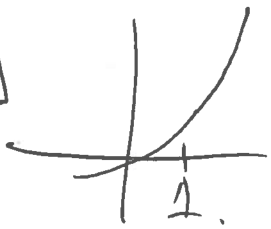
) seems to oscillate.

4.7.14

```
oldx = 2;                             % sets our starting value
f=@(x) exp(x)-3+2*x;      % defines f
fprime=@(x) exp(x)+2;             % defines f'
n=0;                              % start on n=0
nmax=5;                           % max number of steps
tol=.00001;                       % tolerance to stop at;
difference = 1;                   % we will update this variable to be newx-x. start with 1.
xvals = [oldx];                   % store our x values in an array

while  tol<=difference&n<=nmax
    newx = oldx - f(oldx)/fprime(oldx);  % compute new x based on our formula
    xvals = [xvals; newx];                % add new x value to our array
    difference = abs(newx-oldx);         % compute difference to see if we are close
    oldx = newx;                         % continue iterating
    n=n+1;
end

disp(xvals);  %print out xvalues
```



Let's guess our starting value x=2.

```
>> newtonsmethod
    2.0000
    1.1065
    0.6613
    0.5953
    0.5942
    0.5942
```

← converges nicely.

4.8.12] $f(x) = 3e^x + 7\sec^2 x$

$F = 3e^x + 7\tan x + C$

since $\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(\tan x) = \sec^2 x.$

4.8.26] $f' = 2x - 3x^{-4}$ $x > 0,\ f(1) = 3.$

$F = \frac{2x^2}{2} - \frac{3x^{-3}}{-3} = x^2 + x^{-3} + C$

$f(1) = 1 + 1 + C \Rightarrow \underline{C = 1.}$

4.8.34] $f''(t) = 2e^t + 3\sin t$

$f'(t) = 2e^t - 3\cos t + C$

$f(t) = 2e^t - 3\sin t + Ct + D$.

$f(0) = 0 = 2 + D \Rightarrow D = -2$.

$f(\pi) = 0 = 2e^\pi + C\pi - 2$

$$C = \frac{2 - 2e^\pi}{\pi}.$$

4.8.36.

$f'''(x) = \cos x$

$f''(x) = \sin x + C$

$f''(0) = 3 \Rightarrow C = 3$.

$f'(x) = -\cos x + 3x + D$.

$f'(0) = -1 + D = 2$.

$\underline{D = 3}$

$f(x) = -\sin x + \frac{3x^2}{2} + Dx + E,$

$f(0) = 1.$

$f(0) = \quad E = 1$

$\Rightarrow \underline{E = 1}$