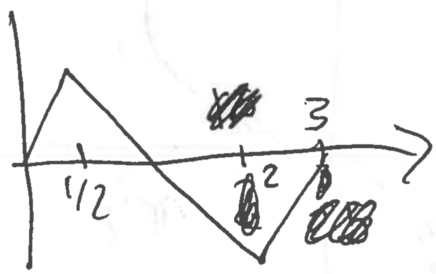
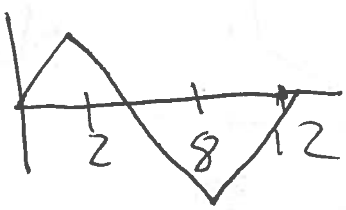
Homework 1 Solutions

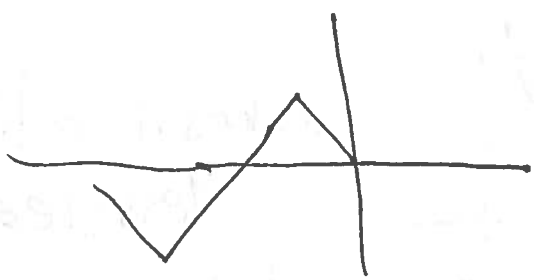
1.3.5) a.) $f(2x)$ = Scale (shrink) horizontally by a factor of 2;



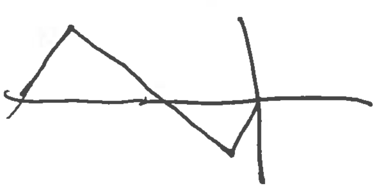
b.) $f(\frac{1}{2}x)$ = stretched horizontally by a factor of 2.



c.) $y=f(-x)$ flipped along y-axis;

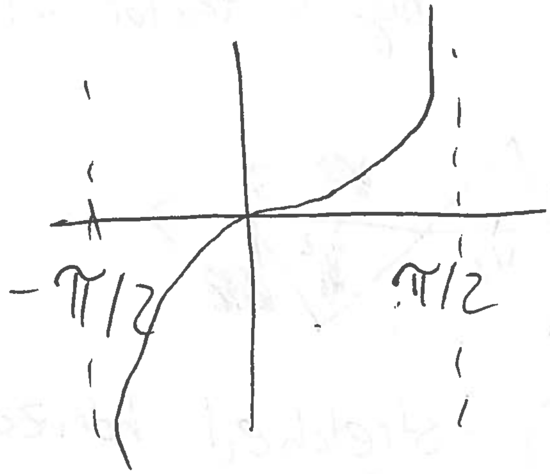


d.) Same as (c) but also flipped on x-axis;

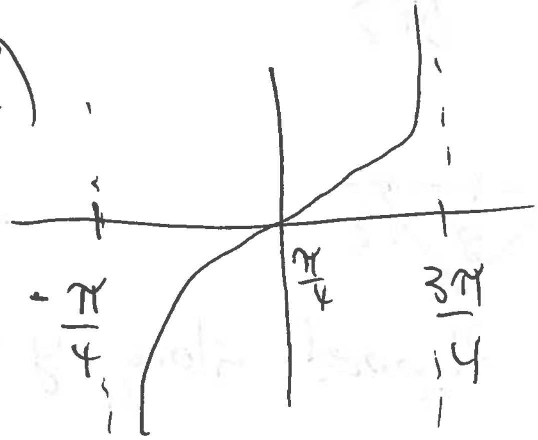


1.3.22) $y = \frac{1}{4}\tan\left(x - \frac{\pi}{4}\right)$.

Start with $f(x) = \tan(x)$.

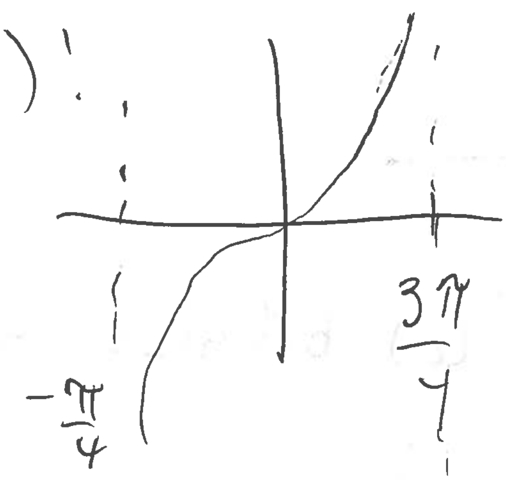


$f\left(x - \frac{\pi}{4}\right)$



Shift right by $\frac{\pi}{4}$

$\frac{1}{4}f\left(x - \frac{\pi}{4}\right)$



Makes it a little less steeeper.

but

$0 \cdot \frac{1}{4} = 0$

and $\infty \cdot \frac{1}{4} = \infty$

so zeroes/asymptotes remain.

1.3.34) $f(x) = \sqrt{x}$, $g(x) = \sqrt[3]{1-x}$.

- $f \circ g = f(g(x)) = \sqrt{\sqrt[3]{1-x}} = \left((1-x)^{1/3}\right)^{1/2}$

$$= (1-x)^{1/6}$$

$$= \sqrt[6]{1-x}.$$

Recall: $\sqrt{x}$ can't take neg. #'s.

$\sqrt[3]{x}$ can take neg. #'s.

Convice yourself!

what is $\sqrt[3]{-27}$?

$\sqrt[4]{x}$ can't.

$\sqrt[5]{x}$ can. → See the pattern?

$\sqrt[6]{1-x}$ can't take negative numbers

so

$1-x \geq 0 \Rightarrow 1 \geq x \Rightarrow x \leq 1$. is the domain.

- $g \circ f = g(f(x)) = \sqrt[3]{1-\sqrt{x}}$ Can't be simplified!

But recall from above: $\sqrt[3]{x}$ has no problems with domain but $\sqrt{x}$ does;

so domain is $x \geq 0$.

$$f \circ f = \sqrt{\sqrt{x}} = x^{1/4}.$$

Again, $x \geq 0$.

$$g \circ g = \sqrt[3]{1 - \sqrt[3]{1-x}}$$

From before, does $\sqrt[3]{\cdot}$ cause any domain problems? No! Domain is $\mathbb{R}$.

1.5.20) a.) $g(t) = \sin(e^{-t})$

$$= \sin\left(\frac{1}{e^t}\right).$$

First, consider $\frac{1}{e^t}$. Are there any problems?

Maybe $\frac{1}{0}$, $e^t = 0$? Can $e^t = 0$? No.

What about sin? Sin's domain is $\mathbb{R}$.

There are no problems. Thus, domain $g(t) = \mathbb{R}$.

b.) $\sqrt{1 - 2^t} = g(t)$.

We know square root can't take negative numbers, thus:

$$1 - 2^t \geq 0 \Rightarrow 1 \geq 2^t.$$

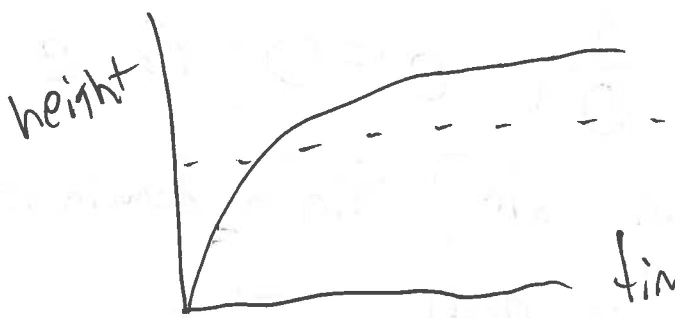
$$\log_2(1) \geq t$$

asks $2^? = 1 \Rightarrow ? = 0$.

$0 \geq t \Rightarrow t \leq 0$ is the domain.

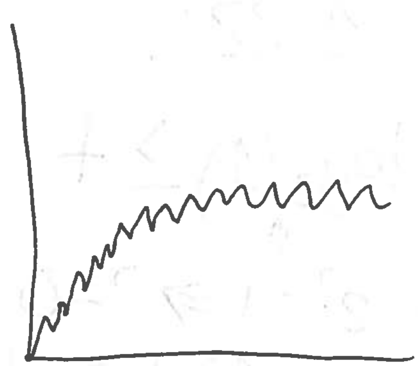
1.6. 18) $f(t)$ = height at age t.

Unless you were in some horrible accident (tasteless, sorry), your height has remained the same or increased over time, meaning it probably looks like:



By the horizontal line test, this is clearly one-to-one.

Note: I guess you could argue that your spine compresses as the day goes on;



would not be one-to-one.

1.6.22) $f(x) = e^{2x-1} = y$.

Recall we want to solve for x!

$$y = e^{2x-1}$$

$$\ln(y) = 2x-1$$

$$\ln(y) + 1 = 2x$$

$$\frac{\ln(y)+1}{2} = x. \Rightarrow f^{-1}(x) = \frac{\ln(x)+1}{2}.$$

1.6.40) By log rules!

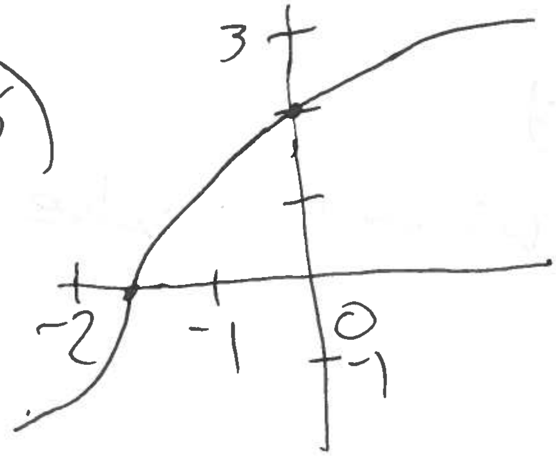
$$\ln(a+b) + \ln(a-b) - 2\ln c =$$

$$\ln(a+b) + \ln(a-b) - \ln c^2 =$$

$$\ln[(a+b)(a-b)] - \ln c^2 =$$

$$\ln\left[\frac{(a+b)(a-b)}{c^2}\right]$$

1.6.18)



- By the horizontal line test, this is definitely one-to-one.
- The domain is a little tricky because the question is unclear (sorry). We know the domain is at least $[-3,3]$ but you can use your imagination and pretend it is $\mathbb{R}$. The range seems to be $[-1,3]$ but it might take on other values outside this range.
- $f^{-1}(2)$ asks what x value gives us $y=2$? It's ~~the~~ $x=0$ from the graph above.
- $f^{-1}(0)$ asks the same question but with crossing the x axis. It's between 2 and 3

1.6.50) a.)

$$\ln(x^2-1)=3$$

$$e^{\ln(x^2-1)}=e^3$$

$$x^2-1=e^3$$

$$x^2=e^3+1$$

$$x=\pm\sqrt{e^3+1}$$

b.) $e^{2x}-3e^x+2=0$.

Note, this problem might be doable via log rules, but the easiest way is to notice that it is a quadratic in e^x. To see this, let:

$z=e^x$, meaning we now have:

$z^2-3z+2=0$, which we can solve via the quadratic equation or factoring.

$$(z-2)(z-1) \Rightarrow z=2 \text{ or } z=1$$

$$e^x=2 \Rightarrow x=\ln(2).$$

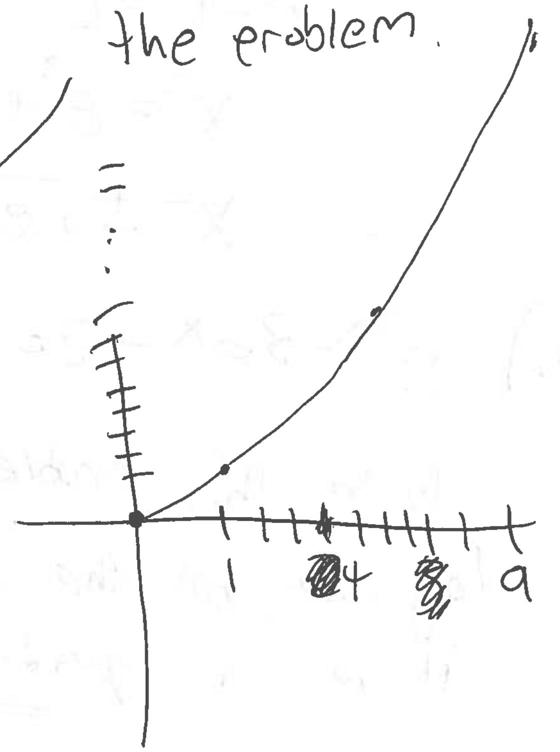
$$e^x=1 \Rightarrow x=\ln(1)=0.$$

1.7.8) $x=t^2, y=t^3$. We can choose some points:

t	x	y
0	0	0
-1	1	-1
-2	4	-8
1	1	-1
2	4	8

I think they forgot to mention $t \geq 0$ in the problem.

t	x	y
0	0	0
1	1	1
2	4	8
3	9	27



Note, to convert to Cartesian, we want:

$y=f(x)$, but we know:

$$y=t^3=(t^2)^{3/2}=x^{3/2}$$

1.7.10) $x=\frac{1}{2}\cos\theta \quad y=2\sin\theta \quad 0\le\theta\le 2\pi$.

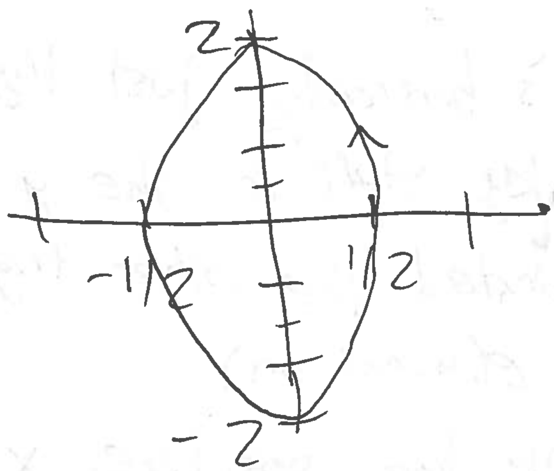
$\cos^2+\sin^2=1$ (trig identity).

$$\Downarrow$$

$$(2x)^2+\left(\tfrac{1}{2}y\right)^2=1$$

$$\frac{x^2}{(1/2)^2}+\frac{y^2}{2^2}=1.$$

Note, this is an <u>elliese</u>!



1.7.26) I know this problem was no longer corrected, but it is doable with some hand-wavy arguments

First, note that only III, II, VI are bounded (do not keeeegoing) so we can conclude they probably involve some trig function.

Note, (f) starts out large and gets smaller as $t \to \infty$, which corresponds to III, as we have a circle that eventually gets smaller and smaller.

VI must be (d), as it's basically just a sin and cos with different periods, so one direction oscillates much faster than the other.

II is (c), as it's basically just trig functions but does funky stuff in the y direction (and we eliminated the other trig graphs by process of elimination)

Note that V only has positive x values and we can try to figure out which equation gives us that. Consider (a) or (b).

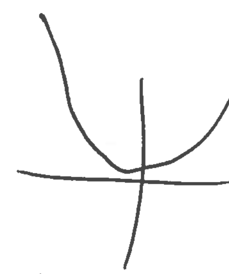
t	x_a	x_b
0	1	0
-1	3	3
1	1	-1
-2	19	8
2	15	0.

Note, (a) always has pos. x values

$\Rightarrow$ V.

and also, (b) goes negative for a little and then stays positive $\Rightarrow$ I.

This leaves only (e) which must be IV but we could realize this by noticing that as $t \to \infty$ sin, cos must be small relative to t and t^2, meaning this graph is roughly:

$x = t, \; y = t^2 \longrightarrow$ 

which is just IV without the wiggles.