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Chers membres du jury,

Rapport sur les travaux présentés par Charles Collot pour le titre de Docteur.

Le dossier présenté par **Charles Collot** se compose de quatre articles. Un d'entre eux est déja accepté dans Mem. Amer. Math. Soc. Les trois autres sont soumis. Je crois que c'est une thèse de très haute qualité et je suis très favorable à la soutenance.

Le document présenté par **Charles Collot** comprend :

chapitre 0 : une introduction avec un énoncé concis des résultats de la thèse,

chapitre 1 : une introduction plus longue qui relie les résultats de la thèse aux avancées récentes sur le sujet. Ca donne aussi une presentation plus detaillee des résultats avec des idees de preuve,

Les chapitres suivants contiennent les preuves rigoureuses. Une introduction et un resumé sont egalement donnés en langue francaise.

The Thesis presented by Charles Collot has six chapters: Chapter 0 is a short presentation of the results presented. Chapter 1 is a long introduction to the subject of blow-up and dynamics near steady states and backward self-similar solutions. It gives a very nice connection between the results presented and the literature and also gives the main ideas of the proofs. The four next chapters (2 to 5) are based on the four papers of the thesis.

In this thesis, Charles Collot studies the dynamic near steady states and backward self-similar solutions for the nonlinear heat and wave equations. The thesis consists of 4 papers :

- Type II blow up manifolds for a supercritical semi-linear wave equation, (to appear to MAMS).
- Non radial type II blow up for the energy supercritical semilinear heat equation
- Dynamics near the ground state for the energy critical nonlinear heat equation in large dimensions, (in collaboration with Merle and Raphaël),
- The stability of type I blow up for the energy supercritical heat equation, (in collaboration with Raphaël and Szeftel)

In the papers "Type II blow up manifolds for a supercritical semi-linear wave equation, (to appear to MAMS)" and " Non radial type II blow up for the energy supercritical semilinear heat equation ", Charles studies type II blow up for the supercritical semi-linear wave and heat equations. These are type II blow up solutions that concentrate the ground state. I will describe the paper about the heat equation. Charles starts by recalling the Joseph-Lundgren exponent p_{JL} which is equal to $+\infty$ in space dimension $d \leq 10$ and $p_{JL} = \frac{1}{d-4-2\sqrt{d-1}}$ if $d \geq 11$. For $d \geq 11$ and $p > p_{JL}$, odd number, Charles constructs a countable family of blow-up solutions with a quantized speed. The proof uses a careful study of the properties of the ground state Q and the linearized equation around it. This study allows Charles to construct a precise approximate solution. He then derives the dynamical system governing the evolution of the parameters of the approximate profile. Then, he uses a bootstrap technique to prove that the exact solution of the equation remains close to the approximate profile.

In the paper "On the stability of type I blow up for the energy supercritical heat equation" (in collaboration with Raphaël and Szeftel), Charles gives the proof of existence of a countable family of type I blow-ups to the supercritical heat equation. First, Charles constructs a countable family of radial solutions to the self-similar equation $\Delta v - \Lambda v + v^p = 0$ when $p_c < p < p_{JL}$. The construction uses a gluing technique. It consists in constructing a family of exterior solutions parametrized by ϵ and a family of interior solutions parametrized by λ and adjusting the two parameters to get a solution on the whole line. To perform the gluing, Charles has to make sure that the function and its derivative are continuous at r_0. Using the implicit function theorem and the continuity at r_0 one deduces that $\epsilon = \epsilon(\lambda)$ has to be a function of λ. The oscillations of the ground state Q allows to make the gluing at a countable possible values of λ. After proving the existence of solutions to the self-similar equation, Charles has to study the linearized operator around these solutions. A major difficulty comes from the fact that these solutions are not very explicit since they were constructed by a gluing technique. To perform this study, Charles had to use spherical harmonics and ODE techniques. The final step of the proof is the dynamical control of the flow. It consists in using a modulation argument to fix the parameters of the approximate solution and a bootstrap argument to control the error term. The properties of the linearized operator are used here to get the right bounds on the error term.

In the paper "Dynamics near the ground state for the energy critical nonlinear heat equation in large dimensions" (in collaboration with Merle and Raphaël), Charles describes all possible dynamics of solutions which are close to the ground state for the energy critical heat equation for $d \geq 7$. There are 3 possible behaviors: a) convergence to a rescaling of the ground state, b) dissipation : $\lim_{t\to\infty} \|u(t)\|_{L^\infty \cap \dot{H}^1} = 0$ or c) type I blow up i.e the ODE type I self-similar behavior with $\|u(t)\|_{L^\infty} \sim (T-t)^{-\frac{d-2}{4}}$. This classification rules out the possibility of type II blow-up near the solitary wave in $d \geq 7$. It is worth noting that R. Schweyer constructed such type of blow up when $d = 4$. This of course shows the importance of the restriction on the dimension in Chalres' result. The proof uses some coercivity estimates on the linearized operator around the ground state. These estimates require the fact that ϵ is orthogonal to the kernel and to the unstable direction Y. It use the fact that $d \geq 7$. Then, Charles introduces a nonlinear modulated decomposition of the solutions near the manifold of solitons : $u = (Q + aY + \epsilon)_{z,\lambda}$. Here $a(t)Y$ is the projection on the unstable direction Y, $z(t)$ is a translation parameter, λ is a scaling parameter and ϵ is a remainder which is orthogonal to the kernel and to Y. The orthogonality condition allows Charles to derive evolution equations for a, ϵ, λ and z. To describe the different possible dynamics, Charles has first to construct two important solutions $Q^{\pm}$ defined on $(-\infty, 0]$ and which are close to Q. He also proves that Q^+ explodes forward in time with type I blow up and that Q^- is global and dissipates : $\lim_{t\to\infty} \|Q^-(t)\|_{L^\infty \cap \dot{H}^1} = 0$. These are the two possible exit behaviors of any solution that starts close to the manifold $\mathcal{M}$ of ground states. To conclude the proof, Charles has to prove that if the solution remains close to $\mathcal{M}$ for all times, then necessary it has to converge to a rescaling of Q otherwise it has to flow one of the exit solutions, namely Q^+ or Q^-.

In conclusion, all these papers are of a very high quality and Charles Collot was able to master many blow up techniques and to understand new difficulties that arise in supercritical problems. I am very favorable to the defense of this thesis.

Sincerely yours,

Masmoudi Nader
Professor