Once more: Do we understand Quantum Mechanics – finally?

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????
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Credits

The pioneers:
Dirac, Heisenberg, Jordan, Schrödinger
Those who (mis?)understood quantum probabilities
Born, Lüders, Schwinger, Wigner
My teachers:
Fierz, Haag, Hepp, Jost, Specker
Other significant people:
Gell-Mann&Hartle, and others
Bannier, Doplicher-Fredenhagen-Roberts
Collaborators:
Pickl, Schilling, Schubnel
A friend who appreciates my efforts:
Blanchard

Outline of Lecture

1. Introduction
2. What is a physical system?
3. Consequences of NC of quantum observables
6. Quantum Probabilities
7. Remarks on relativistic quantum theory

1. Introduction

1.1 Quotations

"In our description of Nature the purpose is not to disclose the 'real essence' of the phenomena but only to track down, so far as it is possible, relations betw. the manifold aspects of our experience."
(Niels Bohr)

"Indeed, space & time are nothing in themselves, but only a certain order of the reality existing & happening in them." (H. Weyl)

"As Riemann pointed out, I believe, the math. cont. is a convenient *fiction* for dealing with phys. phenom. and the math. of ∞ are just a way of approximating (by simplific. through "ideali-zation") an understanding of finite aggregates, whose structures seem too elusive ... for a more direct under-standing ..."

(A. Grothendieck)

"If someone tells you they understand quantum mechanics then all you've learned is that you've met a liar."

(R.P. Feynman)

"Anyone who is not shocked by quantum theory has not understood it."

(N. Bohr)

"We have to ask what it means!"

(K.G. Wilson)

1.2 Purpose of Lecture

- Unified algebraic descr. of class. & quantum-mech. physical systems.
- Clarify what kind of th. of Nature QM is.
- Rôle of causality, emergence of space-time.

Get rid of all the nonsense about QM presently circulating!

2. What is a physical system? Realistic vs. "quantum" ths.

A physical system, S, is specified in terms of phys. quantities, repr. as s.a. linear operators generating a *algebra, $\mathcal{A}_S$

S imbedded in "environm.", E, which it interacts with.

$S \vee E$: "closed" syst., descr. by "kinematical" C^*-alg., $\mathcal{B}_S \supseteq \mathcal{A}_S$, on which time evol. defined.

Fundamental data:

(I) $\mathcal{A}_S \subseteq \mathcal{B}_S$: a C^*-alg. w. $\mathbb{1}$

(II) $\mathcal{S}_S$: states on $\mathcal{B}_S$ ($\to$ on $\mathcal{A}_S$)

(III) $\mathcal{G}_S$: symmetries of S, incl. time evolution, acting as *automorphisms on $\mathcal{B}_S$;

ex.: time evol. $(t,s) \mapsto \alpha_{t,s} \in Aut(\mathcal{B}_S)$.

Def. Algebra of "events" in S:

$\mathring{\mathcal{E}}_S := \langle b \mid b = \prod_k \alpha_{t_k, t_0}(a^k), a^k \in \mathcal{A}_S \rangle$

We assume that $\exists$ type-I C^*-algebra $\mathcal{E}_S = \overline{\mathring{\mathcal{E}}_S}$, with

$$\mathring{\mathcal{E}}_S \subset \mathcal{E}_S \subseteq \mathcal{B}_S$$

$\mathcal{E}_S$: alg. of "possible events" in S – "possible event": When measured, $a_t := \alpha_{t,t_0}(a)$, $a = a^* \in \mathcal{A}_S$, has value in $I \subset \mathbb{R} \leftrightarrow$ spect. proj. $P_{a_t}(I) \in \mathcal{E}_S$.

[(IV) Subsystems/composition of systs., statistics] !

Choice of (I) & (III) depends on equipment available to observe Nature $\to$ observer $\mathcal{O}$.

- New theories arise by "deformation" of (I), (III) (& (IV)).

(I) $\left[\begin{array}{l}\text{cont. ths. of matter} \xrightarrow{k} \text{atomism} \\ \text{class. mechanics} \xrightarrow{h} \text{QM}\end{array}\right.$

(III) Galilei symm. $\xrightarrow{c^{-1}}$ Poincaré $\xrightarrow{R^{-1}}$ de Sitter

(IV) $\left[\begin{array}{l}\text{permutation stat.} \to \text{braid stat.} \\ \text{group symm.} \to \text{quantum groups}\end{array}\right.$

$\swarrow\!\!\!\nearrow$

th. of braided $\otimes$ categories, duality (Tannaka–Krein th.)

Ex. Vlasov th. $\xrightarrow{k}$ Newtonian mech.

$\downarrow h$

wave mechanics

$\Big<$ realistic ("class.") theories R / quantum theories Q

(R) Realistic theories

$\mathcal{B}_S$ abelian $\underset{\text{Gel'fand}}{\Longrightarrow}$ $\mathcal{B}_S \simeq C_0(M_S)$

$M_S = \operatorname{spec} \mathcal{B}_S$ (ex: $M_S = \Gamma$)

$\mathcal{S}_S = \{\text{prob. meas. on } M_S\}$

Pure states $= \delta$-fns. on M_S

$\updownarrow$ $=$ chars of $\mathcal{B}_S$

no superposition principle

no entanglement of S_1 & S_2 in $S_1 \vee S_2$.

*automorphisms of $\mathcal{B}_S$

$\overset{1-1}{\longleftrightarrow}$ homeomorphisms of M_S

Problem. Under what cond. does TM_S exist (is M_S a diff., sympl., ... mf.)?

Assuming TM_S ex., time evol. $\{\alpha_{t,s}\}$ generated by VF, X_t:

$$\dot{\xi}_t = X_t(\xi_t), \ \xi_t \in M_S.$$

→ Realist. & det. descr. of S

$$P_i := \alpha_{t_i,t_0}(\chi_{\Omega_i}) = \chi_{\Omega_i} \circ \phi_{t_i,t_0} = \chi_{\phi^{-1}_{t_i,t_0}(\Omega_i)}$$

($P_i \leftrightarrow$ acquisit. of info. on S at t_i)

Then, for arb. $\xi_0 \in M_S$,

$$\delta_{\xi_0}\Big(\prod_i P_i\Big) = 0 \text{ or } 1!$$

Determinism

Effective dynamics:

$T_{t,s}: \mathcal{S}_S \to \mathcal{S}_S$, w. $T_{t,s} \circ T_{s,u} = T_{t,u}$

→ stoch. processes on M_S

(Q) Quantum theories

$\mathcal{A}_S$, hence $\mathcal{E}_S$ non-abelian

Example: $\mathcal{E}_S$ type-I C^*-alg. (↗Glimm); e.g. group alg. of comp. Lie group (qm spins) or Weyl alg. for S with finite nb. of degs. of freedom.

$\mathcal{Z}_S$: centre of $\mathcal{E}_S$ – abelian, R

$$\overline{\mathcal{E}_S}^{w} \simeq \int^{\oplus}_{\mathrm{spec}\,\mathcal{Z}_S} B(\mathcal{H}_\xi), \quad \xi \in \mathrm{spec}\,\mathcal{Z}_S$$

$\mathcal{H}_\xi$: Hilbert spaces

$\mathcal{S}_S = \{$density matrices on $\mathcal{H}_\cdot\} \otimes$
$\{$prob. meas. on spec $Z_S\}$

Pure states
$= \{$unit rays in $\mathcal{H}_\xi \mid \xi \in \text{spec}\, Z_S\}$

Thus:

- Superposition principle (in $\mathcal{H}_\xi$)
- Entanglement of S_1 & S_2 in $S_1 \vee S_2$.

Dynamics $\{\alpha_{t,s} \in \text{Aut}\, \mathcal{E}_S\}$:
unitary propagators on $\mathcal{H}_\cdot$
$\ltimes$ flows on strata of spec Z_S.

Effective dynamics: Quant.
Markov semigroups –
"Lindbladians".

3. Some Consequences of NC of quantum observables

(i) Preparation of states, indeterminism of QM.

How can S be prepared in specific initial (pure) states, ω, on $\mathcal{E}_S$? (Th. of relaxation to "ground states", "equ. states", … ↗ (B.S.)^2 & J.F., W. DeR & A.K.)

Given ω, one can measure some $a = a^* \in \mathcal{A}_S$ w. $\omega((a-\omega(a))^2) > 0$, even if ω pure $\Rightarrow$

No 0–1 laws, indeterminism!

(ii) "No signaling lemma" (F-P-S)

→ concept of "closed system" is meaningful idealization.

→ ∄ "realistic" interpretation of QM.

(iii) Uncertainty relations

(iv) Kochen-Specker ...

(∄ hidden variables. – Relation to thm. of Kakutani et al.)

(v) Bell $<$'s

(Grothendieck $<$, Tsirelson)

(vi) Aspects of entanglement

Entanglement entropy ...
appl. in quantum info th.

(vii) Quantum marginal problem

(A. Klyachko, M. Christandl et al.)

(viii) Quantum Markov processes

Eff. dynamics (... quantum Brownian motion; Mott tracks; decoherence; ...)
Lindblad generators;
quantum Boltzmann Eq.

4. Quantum Probabilities

(Born, Lüders, Schwinger, Wigner)

$\{P_n, P_{n-1}, \cdots, P_1\}$: time-ordered sequ., "history", of possible events; $P_i \equiv P_i^1 := P_{a^i_{t_i}}(I_i^1)$,

$a^i = (a^i)^* \in \mathcal{A}_S$, $\forall i$, $t_0 < t_1 < \cdots < t_n$,

$P_i^\alpha := P_{a^i_{t_i}}(I_i^\alpha)$, $\alpha = 1, \cdots, k_i$, $I_i^\alpha \cap I_i^\beta = \emptyset$,

$\bigcup_{\alpha=1}^{k_i} I_i^\alpha = \mathbb{R} \Rightarrow \sum_{\alpha=1}^{k_i} P_i^\alpha = \mathbb{1}$;

i.e., $P_i^1, \cdots, P_i^{k_i}$ are complementary possible events.

E, hence $\mathcal{B}_S$, and $\{\alpha_{t,s}\}$ fixed.

QM predicts "frequency"/empir. prob. of $\{P_n, P_{n-1}, \cdots, P_1\}$, given an initial "state" ω on $\mathcal{E}_S$ at t_0.

"Master formula."

$$F_\omega\{P_n, \cdots, P_1\} := \omega(P_1 P_2 \cdots P_{n-1} P_n P_{n-1} \cdots P_2 P_1) \qquad (3)$$

Properties of F_ω :

(i) $F_\omega\{P_n, \cdots, P_1\} \geq 0$

(ii) Set $P_j^1 := P_j$, $P_j^\alpha \cdot P_j^\beta = \delta^{\alpha\beta} P_j^\alpha$,

$\sum_{\alpha=2}^{n_j} P_j^\alpha = \mathbb{1} - P_j$, $n_j \geq 2$.

$$\sum_{\underline{\alpha}} F_\omega\{P_n^{\alpha_n}, \cdots, P_1^{\alpha_1}\} = 1$$

$$\Rightarrow 0 \leq F_\omega\{P_n, \cdots, P_1\} \leq 1 \qquad (4)$$

For "realistic" syst. S, $\mathcal{F}_\omega$ obeys a 0-1 Law if ω is pure.

(iii) "Symm. betw. prediction & retrodiction" – cycl. of tr (A-B-L)

(iv) (Non-)complementarity of "events"

$$\mathcal{F}_\omega\{P_n,\dots,P_j,\dots,P_1\} + \sum_{\alpha=2}^{n_j} \mathcal{F}_\omega\{P_n,\dots,P_j^\alpha,\dots,P_1\}$$

$$\underset{i.g.}{\neq} \mathcal{F}_\omega\{P_n,\dots,P_{j+1},P_{j-1},\dots,P_1\}, \quad (5)$$

unless $j=n$, because, i.g.,

$$\sum_{\alpha\neq\beta} \omega(P_1\cdots P_j^\alpha \cdots P_n \cdots P_j^\beta \cdots P_1) \neq 0 \quad (6)$$

"interference"

$\Rightarrow$ For $n_j > 2$, no meaningful notion of "conditional probability" of P_j, given future; ($\rightarrow$ K-S!)

Thus, i.g. "complementary possible events" do not mut. exclude one another, in QM. (Ex.: Double-slit exp.)

- "δ-consistent histories"

"Evidence" for one of $\{P_j^\alpha\}_{\alpha=1}^{n_j}$ to materialize in meas. of $a^j_{t_j}$, given ω and future events, $P_{j+1}, \dots$:

$$\mathcal{E}^{(j)}_\omega := 1 - \sum_{\substack{1\leq\alpha,\beta\leq n_j \\ \alpha\neq\beta}} |\omega(\cdots P_{j-1} P_j^\alpha P_{j+1} \cdots \cdots P_{j+1} P_j^\beta P_{j-1} \cdots P_1)|$$

$\mathcal{E}^{(j)}_\omega = 1 \Rightarrow$ One of $P_j^1 \cdots P_j^{n_j}$ happens.

...

History $\{P_n, \cdots, P_1\}$ δ-consistent w.r. to ω iff

$$\min_j \mathcal{E}_\omega^{(j)} \geq \delta$$

$\delta \leq 1$ $(0 < 1-\delta \ll 1)$. If $\delta = 1$ history called "consistent": Idealization! δ very close to $1 \Rightarrow \{P_j^\alpha\}_{\alpha=1}^{n_j}$ mut. exclude each other, FAPP, $\forall j$.

- Let

$$H_j := \Big(\prod_{i=j+1}^{n} P_i\Big)\Big(\prod_{i=n}^{j+1} P_i\Big), \ j = 1, \cdots, n,$$

where $\{P_n, \cdots, P_1\}$ a history.

Lemma. If

$$\|[P_j, H_j]\| < \varepsilon, \ \forall j = 1, \cdots, n-1,$$

ε small enough, then $\exists$ $\{\tilde{P}_n, \cdots, \tilde{P}_1\}$, with $\|\tilde{P}_i - P_i\| < C_n \varepsilon$,

$$[\tilde{P}_j, \tilde{H}_j] = 0,$$

$\tilde{H}_j$ are orth. projections.

$\longrightarrow$ In the vicinity of δ-consistent histories there are consistent hists.!

- Generation of δ-consistent histories: Dephasing & Decoherence! sep. lecture.

E $\rightarrow$

A remark on decoherence

For $b \in \mathcal{E}_S$ given by

$$b = \prod_k \underbrace{\alpha_{t_k, t_0}(a^k)}_{\equiv a^k_{t_k}}, \quad a^k \in \mathcal{A}_S,$$

define

$$\tau_t(b) := \prod_k \alpha_{t_k + t, t_0}(a^k).$$

→ Defines τ_t on $\mathcal{E}_S$.

Def. E induces "decoherence" in measurement of $a^j_{t_j} = (a^j_{t_j})^*$ in $\mathcal{A}_S$ iff

$$[a^j_{t_j}, \tau_t(b)] \xrightarrow[t\to\infty]{w} 0, \qquad \text{(AC)}$$

$\forall\, b \in \mathcal{E}_S$.

Given an arb. state, ω, on $\mathcal{E}_S$ ($\subseteq \mathcal{B}_S$ "$= \mathcal{A}_{S \vee E}$"), cond. (AC) (= "asympt. centrality") implies that

$$\omega(\tau_t(b)) \xrightarrow[t\to\infty]{} \sum_{\alpha=1}^{k_j} \omega(P_j^\alpha \tau_t(b) P_j^\alpha),$$

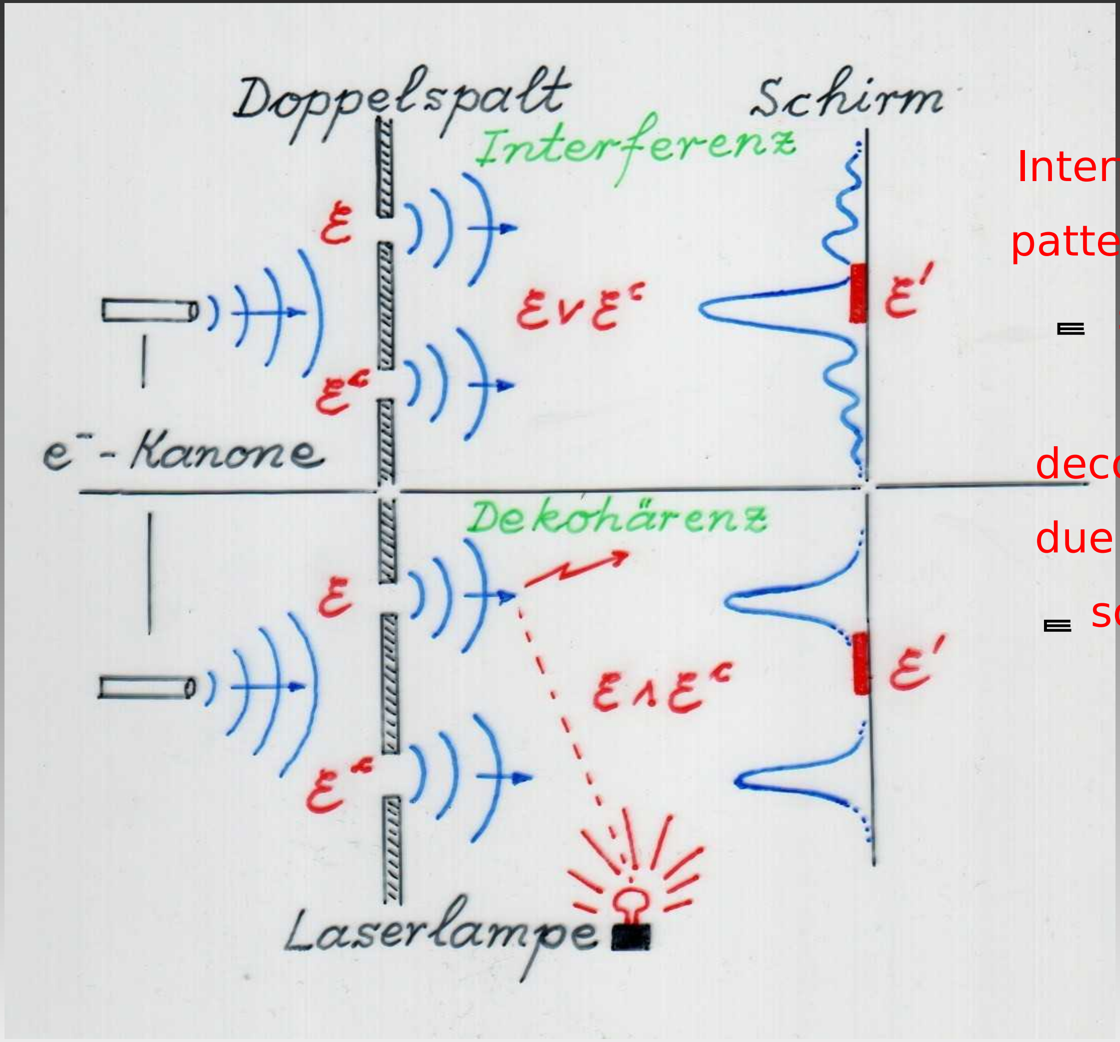
i.e., interference terms vanish, as time, t, elapsing betw. j^{th} and subsequent measurements tends to ∞.

[Weaker cond. is dephasing.]

Choice of E & $\alpha_{t,s}$ is crucial; (ex.: double slit).

An illustration of interference and decoherence in the double-slit experiment

Particle gun =



Interference pattern on = screen

decoherence due to light = scattering

- Problems with conditional probabilities:

$$\mathcal{F}_\omega\{P_n \cdots | P_j | \cdots P_1\}$$

$$\overset{?}{:=} \begin{cases} * \quad \dfrac{\mathcal{F}_\omega\{\cdots P_j \cdots\}}{\mathcal{F}_\omega\{\cdots P_j \cdots\} + \mathcal{F}_\omega\{\cdots P_j^{\perp} \cdots\}}, \text{ or} \\ \qquad\qquad \neq \\ \dfrac{\mathcal{F}_\omega\{\cdots P_j \cdots\}}{\sum_{\alpha=1}^{n_j} \mathcal{F}_\omega\{P_n \cdots P_j^{\alpha} \cdots P_1\}}, \text{ or} \\ \qquad\qquad \neq \\ \dfrac{\mathcal{F}_\omega\{\cdots P_j \cdots\}}{\mathcal{F}_\omega\{\cdots P_{j-1} P_{j+1} \cdots\}} \end{cases}$$

$\uparrow$

$\not\leq 1$, i.g.

- An observation ~ K-S

If $n_j = 2, \forall j$, (only binary quantities are measured) then

* defines cond. probs. unambiguously.

$\hookrightarrow$ class. prob. theory

- δ- consistent histories, including $\delta = 1$, do i.g. not obey "0-1 laws" even if ω is pure; (and $\omega|_{\mathcal{A}_S}$ pure).

5. Remarks on relativistic Quantum Theory

Algebras $\mathcal{A}_S, \mathcal{E}_S, \dots \longrightarrow$

$\mathcal{A}_{(S,O)}, \mathcal{E}_{(S,O)}, \dots, \quad O \in \Omega$

("all observers of S").

$\Delta \equiv \Delta_{(P,O)}$ interval of proper times of O during which O has observed event $P \in \mathcal{E}_{(S,O)}$

$$\Rightarrow \quad \mathcal{E}_{(S,O)} = \overline{\bigvee_{\Delta \subset \mathcal{L}_O} \mathcal{E}_{(S,O)}(\Delta)}$$

$P_1 \underset{O}{<} P_2$ iff $\Delta_{(P_1,O)} < \Delta_{(P_2,O)}$

If $P_1 \underset{O}{<} P_2$, $\forall\, O \in \Omega$,

then "P_1 in the past of P_2":

$$P_1 < P_2$$

If, however, order of P_1 & P_2 differs for <u>some</u> O & $O' \neq O$ in Ω then $P_1 \times P_2$. Then consistency of quantum probabilities $\Rightarrow$

$$[P_1, P_2] = 0\,!$$

Given P_1, P_2, with $P_1 < P_2$, define

$$\mathcal{E}_{(P_1,P_2)} := \langle P \mid P_1 < P < P_2 \rangle$$

$$\mathcal{E}^c_{(P_1,P_2)} := \langle P \mid P \times P',\ \forall P' \in \mathcal{E}_{(P_1,P_2)} \rangle$$

III

→ Nets of "local algebras"

U. Bannier: From such a net
one can reconstruct a
Hausdorff space, $\mathcal{M}$, w.
causal structure:
space-time

Type of $\mathcal{E}_{(S,0)}(\Delta)$ in RQFT:
hyperfinite $\underline{III}_1$

Rôle of Huyghens' principle (D. Buchholz) in gen. of "consistent hists.", …

Thank you!

Indeed, thanks
for your patience!
I hope you enjoyed
this talk!