Stochastic calculus, homework 7, due November 14th.

Exercise 1. Let $X_t = \int_0^t (\sin s)\mathrm{d}B_s$. Prove that this is a Gaussian process. What are $\mathbb{E}(X_t)$ and $\mathbb{E}(X_s X_t)$? Prove that

$$X_t = (\sin t)B_t - \int_0^t (\cos s) B_s \mathrm{d}s.$$

Exercise 2. Prove that if f is a deterministic continuous square integrable function,

$$\mathbb{E}\left(B_t \int_0^\infty f(s)\mathrm{d}B_s\right) = \int_0^t f(s)\mathrm{d}s.$$

Exercise 3. Assume f is a deterministic continuous function. What is the limit in probability of $\frac{1}{B_t}\int_0^t f(s)\mathrm{d}B_s$ as $t \to 0$?