Probability, homework 7, due November 8th.

Some exercises are from *A first course in probability*, ninth edition, by Sheldon Ross.

Exercise 1. Let X be a random variable with probability density function $f x) = c(1 - x^2)$ for $-1 < x < 1$, 0 otherwise.
 (a) What is the value of c?
 (b) What is the cumulative distribution function of X?

Exercise 2. The density function of X is given by $f(x) = a + bx^2$ for $0 < x < 1$, 0 otherwise. If $\mathbb{E}(X) = 3/5$, find a and b. What if $\mathbb{E}(X) = 6/5$?

Exercise 3. A point is chosen at random on a line segment of length L. Interpret this statement, and find the probability that the ratio of the shorter to the longer segment is less than $1/4$.

Exercise 4. Let X be a normal random variable with mean 12 and variance 4. Find the value of c such that $\mathbb{P}(X > c) = 1/10$.

Exercise 5. The number of years a radio functions is exponentially distributed with parameter $\lambda = 1/8$. If Jones buys a used radio, what is the probability that it will be working after an additional 8 years?

Exercise 6. Let X have probability density f. Find the probability density function of the random variable Y defined by $Y = aX + b$.

Exercise 7. Show that if Z is a standard normal random variable; then, for $x > 0$,
 (a) $\mathbb{P}(Z > x) = \mathbb{P}(Z < -x)$;
 (b) $\mathbb{P}(|Z| > x) = 2\mathbb{P}(Z > x)$;
 (c) $\mathbb{P}(|Z| < x) = 2\mathbb{P}(Z < x) - 1$.

Exercise 8. Assume X has density f, a function supported on $\mathbb{R}_+$. What is the density of $X/(1 + X)$?

Exercise 9. Let X be a Cauchy random variable. What is the density of $1/X$?

Exercise 10. Let X and Y be two independent random variables such that $X + Y$ has the same density as X. What is Y?