Probability, homework 5, due October 11th.

From *A first course in probability*, ninth edition, by Sheldon Ross.

Exercise 1. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. What are the possible values of X? For $n = 3$, if the coin is assumed fair, what are the probabilities associated with the values that X can take on?

Exercise 2 A salesman has scheduled two appointments to sell encyclopedias. His first appointment will lead to a sale with probability 0.3, and his second will lead independently to a sale with probability 0.6. Any sale made is equally likely to be either for the deluxe model, which costs \$1000, or the standard model, which costs \$500. Determine the probability mass function of X, the total dollar value of all sales.

Exercise 4 The National Basketball Association (NBA) draft lottery involves the 11 teams that had the worst wonlost records during the year. A total of 66 balls are placed in an urn. Each of these balls is inscribed with the name of a team: Eleven have the name of the team with the worst record, 10 have the name of the team with the secondworst record, 9 have the name of the team with the thirdworst record, and so on (with 1 ball having the name of the team with the 11th-worst record). A ball is then chosen at random, and the team whose name is on the ball is given the first pick in the draft of players about to enter the league. Another ball is then chosen, and if it belongs to a team different from the one that received the first draft pick, then the team to which it belongs receives the second draft pick. (If the ball belongs to the team receiving the first pick, then it is discarded and another one is chosen; this continues until the ball of another team is chosen.) Finally, another ball is chosen, and the team named on the ball (provided that it is different from the previous two teams) receives the third draft pick. The remaining draft picks 4 through 11 are then awarded to the 8 teams that did not win the lottery, in inverse order of their wonlost records. For instance, if the team with the worst record did not receive any of the 3 lottery picks, then that team would receive the fourth draft pick. Let X denote the draft pick of the team with the worst record. Find the probability mass function of X.

Exercise 5 Four buses carrying 148 students from the same school arrive at a football stadium. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let X denote the number of students who were on the bus carrying the randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on her bus.

(a) Which of $\mathbb{E}[X]$ or $\mathbb{E}[Y]$ do you think is larger? Why?

(b) Compute $\mathbb{E}[X]$ and $\mathbb{E}[Y]$.

Exercise 6 A satellite system consists of n components and functions on any given day if at least k of the n components function on that day. On a rainy day, each of the components independently functions with probability p_1, whereas on a dry day, each independently functions with probability p_2. If the probability of rain tomorrow is α, what is the probability that the satellite system will function?

Exercise 7 If $\mathbb{E}[X] = 1$ and $\mathrm{Var}(X) = 5$, find
(a) $\mathbb{E}[(3 - X)^2]$;
(b) $\mathrm{Var}(3 + 4X)$.

Exercise 8 If X has distribution function F, what is the distribution function of e^X?

Exercise 9 Let X be a binomial random variable with parameters n and p. Show that
$$\mathbb{E}\left(\frac{1}{1+X}\right) = \frac{1-(1-p)^{n+1}}{(n+1)p}.$$

Exercise 10 Show that X is a Poisson random variable with parameter λ, then $\mathbb{E}[X^n] = \lambda\mathbb{E}[(X+1)^{n-1}]$ Use this result to compute $\mathbb{E}[X^3]$.