Probability, homework 2, due September 20th.

From *A first course in probability*, ninth edition, by Sheldon Ross.

Exercise 1. Two dice are thrown. Let E be the event that the sum of the dice is odd, let F be the event that at least one of the dice lands on 1, and let G be the event that the sum is 5. Describe the events EF, $E \cup F$, FG, EF^c, and EFG.

Exercise 2. In an experiment, die is rolled continually until a 6 appears, at which point the experiment stops. What is the sample space of the experiment? Let E_n denote the event that n rolls are necessary to complete the experiment. What points in the sample space are contained in E_n? What is

$$(\cup_{n=1}^{\infty} E_n)^c \text{ ?}$$

Exercise 3. A hospital administrator codes incoming patients suffering gunshot wounds according to whether they have insurance (coding 1 if they do and 0 if they do not) and according to their condition, which is rated as good (g), fair (f), or serious (s). Consider an experiment that consists of the coding of such a patient.

(i) Give the sample space of this experiment.
(ii) Let A be the event that the patient is in serious condition. Specify the outcomes in A.
(iii) Let B be the event that the patient is unin- sured. Specify the outcomes in B.
(iv) Give all the outcomes in the event $B^c \cup A$.

Exercise 4. Prove that

$$(\cup_1^{\infty} E_i)\, F = \cup_1^{\infty} E_i F.$$

Exercise 5. Let E, F, and G be three events. Find expressions for the events so that, of E, F, and G,

(i) only E occurs;
(ii) both E and G, but not F, occur;
(iii) at least one of the events occurs;
(iv) at least two of the events occur;
(v) all three events occur.

Exercise 6. Suppose that A and B are mutually exclusive events for which $\mathbb{P}(A) = 0.3$ and $\mathbb{P}(B) = 0.5$. What is the probability that

(i) either A or B occurs?
(ii) A occurs but B does not?
(iii) both A and B occur?

Exercise 7. If N people, including A and B, are randomly arranged in a line, what is the probability that A and B are next to each other? What is the people were arranged in a circle?

Exercise 8. An urn contains n red and m blue balls. They are withdrawn on at a time until a total of r ($r \leq n$) red balls have been withdrawn. Find the probability that a total of k balls are withdrawn.