Probability, homework 4, due October 7.

Exercise 1. Suppose that 3 percent of men and .5 percent of women are color blind. A color-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females. What if the population consisted of twice as many males as females?

Exercise 2. There are 3 coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased coin that comes up heads 65 percent of the time. When one of the 3 coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin?

Exercise 3. Suppose a distribution function F is given by

$$F(x) = \frac{1}{4}\mathbb{1}_{[0,\infty)}(x) + \frac{1}{2}\mathbb{1}_{[1,\infty)}(x) + \frac{1}{4}\mathbb{1}_{[2,\infty)}(x)$$

What is the probability of the following events, $(-1/2, 1/2)$, $(-1/2, 3/2)$, $(2/3, 5/2)$, $(3, \infty)$?

Exercise 4. let X have a geometric distribution with parameter $p \in (0,1)$ ($\mathbb{P}(Y = k) = p^k(1-p)$, $k = 0, 1, 2, \dots$). Show that for $r = 2, 3, \dots$ we have

$$\mathbb{E}(X(X-1)\dots(X-r+1)) = \frac{r!p^r}{(1-p)^r}.$$

Exercise 5. Let X have a binomial distribution with parameters (p, n). Prove that X is even with probability

$$\frac{1}{2}\left(1 + (1-2p)^n\right).$$

Exercise 6. Let $g : [0, \infty) \to [0, \infty)$ be strictly increaing and nonnegative. Show that

$$\mathbb{P}\left(|X| \geq a\right) \leq \frac{\mathbb{E}(g(|X|))}{g(a)}.$$