Complex analysis, homework 8 due March 28th.

Exercise 1. [6 points] Let C be the arc defined by

$$z(t) = \begin{cases} \pi e^{i\pi t} & \text{if } 0 \leq t \leq 1, \\ -\pi + i(t-1)\ln(2) & \text{if } 1 \leq t \leq 2, \end{cases}$$

and $f(z) = \cos(z)\sin^2(z)$. Calculate the following integral (give your answer in $x + iy$ form)

$$\int_C f(z)\,\mathrm{d}z.$$

Exercise 2. [6 points] Let $z_0 \in \mathbb{C}$ and $r > 0$. Let C be the positively oriented circle of radius r about z_0 given by

$$z(\theta) = z_0 + re^{i\theta}, \quad 0 \leq \theta \leq 2\pi.$$

Evaluate the following integral (give your answer in terms of z_0)

$$\int_C \frac{z+i}{z-z_0}\,\mathrm{d}z.$$

Exercise 3. [6 points] Let C be a closed contour. Let f be a piecewise continuous function on C. Prove that the integral $\int_C f(z)\,\mathrm{d}z$ does not depend of the choice of the initial point of the contour. More precisely, assume C is given by $z = z(t)$, $a \leq t \leq b$, fix some $t_0 \in [a,b]$ and define C' by

$$z = w(t) = \begin{cases} z(t) & \text{if } t_0 \leq t \leq b, \\ z(t-b+a) & \text{if } b \leq t \leq b-a+t_0, \end{cases}$$

Then you have to prove $\int_C f(z)\,\mathrm{d}z = \int_{C'} f(z)\,\mathrm{d}z$.

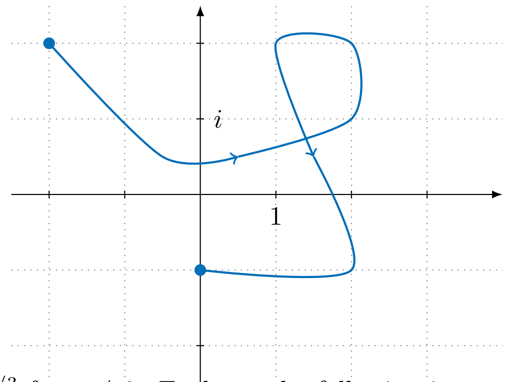
Exercise 4. [6 points] Let C be the arc defined by

$$z(t) = \begin{cases} it & \text{if } 0 \leq t \leq 1, \\ i + (t-1) & \text{if } 1 \leq t \leq 2, \\ 1 + i - i(t-2) & \text{if } 2 \leq t \leq 3, \\ 1 - (t-3) & \text{if } 3 \leq t \leq 4. \end{cases}$$

Evaluate the following integral (give your answer in $x + iy$ form)

$$\int_C \frac{e^{z^2}}{z^2+4}\,\mathrm{d}z$$

Exercise 5. [6 points] Let C be the following contour (its exact definition does not matter but some of its properties do):



Let $f(z) = \text{P.V.}\, z^{1/3}$ for $z \neq 0$. Evaluate the following integral (give your answer in $x + iy$ form)

$$\int_C f(z)\,\mathrm{d}z.$$