Theory of Numbers homework 7
Euler’s phi function, Euler’s theorem, RSA
Due November 5th, 2018

Please hand in your homework stapled, with your name written on it. All answers have to be justified, and details of calculations must be given.

In all what follows, $\phi$ denotes Euler’s phi function.

**Exercise 1.** Establish each of the assertions below:

1. If $n$ is an odd integer, then $\phi(2n) = \phi(n)$.
2. $\phi(3n) = 2\phi(n)$ if and only if 3 does not divide $n$.
3. $\phi(n) = \frac{n}{2}$ if and only if $n = 2^k$ for some $k \geq 1$.

**Exercise 2.** Show that there are infinitely many integers $n$ such that $\phi(n) = \frac{n}{3}$.

**Exercise 3.** Describe the integers $n$ such that $\phi(n)$ is not divisible by 4.

**Exercise 4.** Find the remainder of $11^{1213}$ in the Euclidean division by 26.

**Exercise 5.** Find the last two digits of $2^{2018}$.

**Exercise 6.** You want to do RSA cryptography with $n = pq$ where $p = 11$ and $q = 13$.

1. What is the smallest possible value of the encode exponent $e$ that you can choose?
2. Taking $e$ to be equal to this value, compute a decode exponent $d$.
3. Bob wants to let Alice know on which day in October your Theory of Numbers midterm is. What message will he be sending to Alice?